

1.1

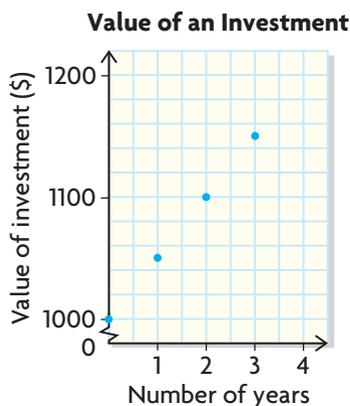
Simple Interest

YOU WILL NEED

- calculator
- graph paper
- straightedge

EXPLORE...

- An amount of money was invested. Interpret the graph below to determine
 - a) how much money was invested,
 - b) the value of the investment at the end of the first year, and
 - c) the rate at which the value grew between the beginning and end of the first year.



term

The contracted duration of an investment or loan.

interest

The amount of money earned on an investment or paid on a loan.

fixed interest rate

An interest rate that is guaranteed not to change during the term of an investment or loan.

principal

The original amount of money invested or loaned.

GOAL

Solve problems that involve simple interest.

INVESTIGATE the Math

Sera is 20 years old and needs money to pay for college. When she was born, her grandparents bought her a \$500 Canada Savings Bond (CSB) with a **term** of 10 years. They chose a CSB as an investment because they liked the security of loaning money to the government. The **interest** earned was determined using a **fixed interest rate** of 6% per year on the original investment and was paid at the end of each year until Sera's 10th birthday.



? How can you determine the current value of Sera's CSB?

- A.** How much interest was earned on the **principal** by the end of the first year?

- B. Determine the **simple interest** earned each year, the accumulated interest, and the value of the investment for the first 4 years. Organize your calculations in a table like the one below.

Year	Value of Investment at Start of Year (\$)	Simple Interest Earned Each Year (\$)	Accumulated Interest (\$)	Value of Investment at End of Year (\$)
0				500
1	500			
2				
3				
4				

- C. Is the simple interest earned each year constant or variable? Explain.
- D. Describe the relationship between the number of years, the interest earned each year, and the accumulated interest.
- E. Use the relationship from part D to predict the value of the investment after 10 years.
- F. Graph the growth of the investment until its **maturity** at 10 years using “Time (years)” as the domain and “Value of the investment (\$)” as the range. Is your prediction in part E supported by your graph?

Reflecting

- G. Describe your graph. What does the shape of your graph tell you about the type of growth?
- H. What do the y -intercept and slope represent for the investment?
- I. Describe how the value of the investment would change at maturity in each situation below, compared with the original situation. Explain why.
- If the principal was \$1000, but the interest was still 6% for 1 term of 10 years
 - If the principal was \$500, but the interest was 5% for 1 term of 10 years
 - If the principal was \$500 and the interest was 6%, but for 2 terms of 10 years

simple interest

The amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate.

maturity

The contracted end date of an investment or loan, at the end of the term.

Communication **Tip**

Interest rates are communicated as a percent for a time period. Since most often the time period is per year or **per annum** (abbreviated as **/a**), a given percent is assumed to be annual unless otherwise stated. For example, an interest rate of 4% means 4%/a or 4% interest per year.

APPLY the Math

EXAMPLE 1

Solving a simple interest problem

Marty invested in a \$2500 guaranteed investment certificate (GIC) at 2.5% simple interest, paid annually, with a term of 10 years.

- How much interest will accumulate over the term of Marty's investment?
- What is the **future value** of his investment at maturity?
- Use Marty's investment to write an algebraic expression that could be used to determine the future value of any investment earning simple interest.

future value

The amount, A , that an investment will be worth after a specified period of time.

Marty's Solution

- a) Amount of interest that will accumulate in 1 year:

$$(0.025)(2500) = 62.5$$

I multiplied the interest rate (as a decimal) by the principal.

In 1 year, \$62.50 in interest will accumulate.

Since simple interest earned is a constant amount, and the simple interest earned in 1 year is \$62.50, 10 times that amount will accumulate in 10 years.

$$(62.5)(10) = 625$$

In 10 years, \$625 in interest will accumulate.

$$i = Prt$$

I noticed that the accumulated interest was the product of the principal, the interest rate, and the number of years.

- b) Principal: \$2500

Accumulated interest: \$625

Future value of investment:

$$\$2500 + \$625 = \$3125$$

I knew that the future value at maturity is the sum of the principal and the accumulated interest, over the full term.

- c) Let A represent the amount, or future value of the investment,
 P represent the principal amount invested,
 t represent the time in years, and
 r represent the interest rate per annum:

$$A = P + Prt$$

$$A = P(1 + rt)$$

I wrote an equation to show that the future value or amount (A) is the sum of the principal (P) and the accumulated interest (Prt).

I factored the expression on the right. I can use either of the equivalent forms of the equation.



Your Turn

How would an interest rate of 3.5% change the future value of Marty's investment?

EXAMPLE 2

Representing the growth of a simple interest investment

Sunni invested \$15 000 in a savings account. Sunni earned a simple interest rate of 8%, paid semi-annually on her investment. She intends to hold the investment for 4.5 years, when she will withdraw all the money to buy a car. Determine the value of the investment at each half year until she withdraws the money.



Sunni's Solution: Using a table of values

Principal, P , is \$15 000.

Rate, r , is 8%, or 0.08.

Time, t , is 0, 0.5, 1, 1.5, ..., 4.5 years.

Year	Value of Investment (\$)
0	15 000
0.5	$15\,000 + 600 = 15\,600$
1.0	$15\,000 + 1200 = 16\,200$
1.5	$15\,000 + 1800 = 16\,800$
2.0	$15\,000 + 2400 = 17\,400$
2.5	18 000
3.0	18 600
3.5	19 200
4.0	19 800
4.5	20 400

Since interest is paid semi-annually, I knew that I could withdraw money after any half-year period and earn the interest up to that time.

I set up a table with a row for each half year. I determined the value of the investment at the end of each half year by adding the principal and the accumulated interest ($P + Prt$). For the first half year, this was $15\,000 + 15\,000(0.08)(0.5)$ or $15\,000 + 600$.

After calculating the value up to 2 years, I noticed a pattern: the investment grew by a constant amount, \$600 every half year. I used the pattern to determine the value for each half-year interval until 4.5 years.

$$A = P + Prt$$

$$A = 15\,000 + (15\,000)(0.08)(4.5)$$

$$A = 20\,400$$

I verified the value at 4.5 years using the simple interest formula.

Taj's Solution: Using a graph

$$A = P + Prt$$

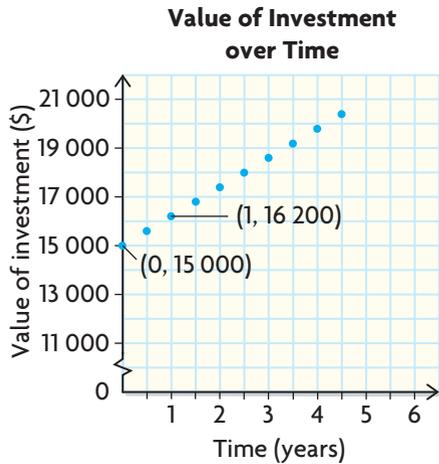
$$A = 15\,000 + (15\,000)(0.08)t$$

$$A = 15\,000 + 1200t$$

$$A = 1200t + 15\,000$$

The A -intercept is 15 000.

The slope is 1200.



The value of the investment will be \$20 400 after 4.5 years.

I substituted the given information into the formula for future value of a simple interest investment.

I noticed that the equation of the line was in the form $y = mx + b$.

I knew that b or P was the A -intercept (\$15 000) and m or Pr was the slope (1200). I also knew that I needed only two points to graph a linear relation.

Because interest is earned only at the end of each half-year period, t is not continuous in this situation, so the graph had to be a series of discrete points.

I created a grid and plotted the principal at (0, 15 000). I used the slope to determine a second point, (1, 16 200).

Since the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ is 1200, there is a rise of \$1200 for every run of 1 year.

I used my ruler to extrapolate the remaining eight points, one at each half-year interval.

Your Turn

How did the formula for the future value of a simple interest investment help Sunni create the table? How did it help Taj create the graph?

EXAMPLE 3

Determining the duration of a simple interest investment

Ingrid invested her summer earnings of \$5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

rate of return

The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

- How long will it take for the future value of the investment to grow to \$8000?
- What is Ingrid's **rate of return**?



Ingrid's Solution

a) $A = P + Prt$

P is \$5000.

r is 8%, or 0.08.

A is \$8000.

$$8000 = 5000 + (5000)(0.08)t$$

$$3000 = 400t$$

$$7.5 = t$$

I knew P , r , and A . I determined t by substituting these known values into the formula $A = P + Prt$ and solving for t .

Because I needed to isolate t , I knew that the $A = P + Prt$ form of the equation would have fewer solution steps than the $A = P(1 + rt)$ form would.

It will take 8 years for the future value of the investment to be at least \$8000.

I knew 7.5 years would not work because the interest is paid annually. This meant that I had to round up to the next whole year. It also meant that, at 8 years, the future value would be more than \$8000.

b) After 8 years:

$$A = P + Prt$$

$$A = 5000 + (5000)(0.08)(8)$$

$$A = 8200$$

At 8 years, the future value will be \$8200.

Interest earned:

$$\$8200 - \$5000 = \$3200$$

I determined the interest earned by subtracting the principal from the future value.

$$\text{Rate of return} = \frac{3200}{5000}$$

$$\text{Rate of return} = 0.64$$

I compared the interest earned with the principal to determine the rate of return.

The rate of return is 64% over 8 years.

Your Turn

How would each situation below change the length of time needed for the future value of Ingrid's investment to grow to \$8000 under simple interest conditions?

- If Ingrid invested principal of only \$4000 at 8%, paid quarterly
- If Ingrid earned 4% on principal of \$5000
- If 8% interest on principal of \$5000 was paid semi-annually

EXAMPLE 4**Determining the rate of interest on a simple interest investment**

Grant invested \$25 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.

- If the future value of the CSB is \$29 375 at the end of 5 years, what interest rate does the CSB earn?
- Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

Cilla's Solution

a) $A = P + Prt$

A is \$29 375.

P is \$25 000.

t is 5 years.

$$29\,375 = 25\,000 + (25\,000)(r)(5)$$

$$4375 = 125\,000r$$

$$r = 0.035$$

The interest rate was 3.5%.

I chose the $A = P + Prt$ form of the equation because I needed to isolate r . I knew that this form of the equation would have fewer solution steps.

I determined r by substituting the known values into the formula and solving for r .

The annual interest rate was expressed as a decimal, so I changed it to a percent.

I knew that the interest rate was an annual rate.

b) $A = P(1 + rt)$

P is \$25 000.

r is 0.035.

t is 4 years.

$$A = 25\,000(1 + (0.035)(4))$$

$$A = 28\,500$$

Grant had \$28 500 for a down payment.

I used a value of 4, instead of 4.5, for t because the interest is paid annually. Since Grant cashed in the CSB after 4.5 years, he did not earn interest for the last half year.

Your Turn

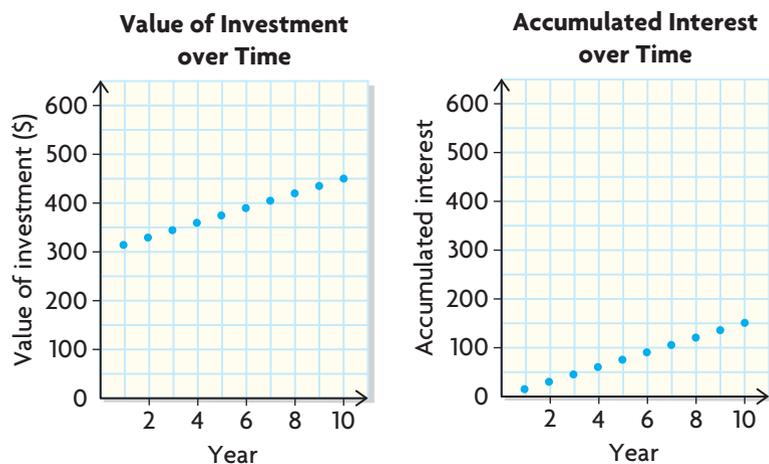
How would the interest rate change in each situation? Explain.

- If Grant invested principal of \$20 000 instead, and the CSB grew to \$29 375 in 5 years
- If it took 8 years for Grant's principal of \$25 000 to grow to \$29 375

In Summary

Key Ideas

- Simple interest is determined only on the principal of an investment.
- The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest earned over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. For example, the following graphs show principal of \$300 invested at 5% interest, paid annually, over a term of 10 years.



Need to Know

- The amount of simple interest earned on an investment can be determined using the formula

$$I = Prt$$

where I is the interest, P is the principal, r is the annual interest rate expressed as a decimal, and t is the time in years.

- The future value or amount, A , of an investment that earns simple interest can be determined using the formula

$$A = P + Prt$$
$$\text{or } A = P(1 + rt)$$

where P is the principal, r is the interest rate expressed as a decimal, and t is the time in years.

- Unless otherwise stated, an interest rate is assumed to be annual, or per annum.
- Even though interest rates are usually annual, interest can be paid out at different intervals, such as annually, semi-annually, monthly, weekly, and daily.

CHECK Your Understanding

1. Determine the future value of each investment if it earns simple interest.
 - a) 8-year term on a principal of \$30 000 at 3.5%
 - b) 1.25% interest paid quarterly for 4 years on \$10 000
 - c) 0.5% interest paid weekly for 2 years on \$25 000
 - d) 20-year term at 7.4% on a deposit of \$12 000
2. Cam has \$5000 to invest. He wants his principal to grow to \$6500 in 5 years so that he can afford a new drum kit.
 - a) What simple interest rate will allow him to meet his goal?
 - b) Suppose that interest is paid semi-annually and Cam withdraws all the money after 3.25 years. How much money will he have?



3.
 - a) Principal of \$1000 is invested at 5% simple interest, paid annually, for 5 years. What is the rate of return?
 - b) Which option below would yield the greatest future value? What is the rate of return for this option?
 - A. increasing the principal to \$1050
 - B. increasing the interest rate to 6%
 - C. paying interest every 6 months
 - D. increasing the term to 6 years

PRACTISING

4. Char invested \$4000 at a simple interest rate of 2.3%.
 - a) What is the value of her investment after 5 years?
 - b) What is its value after 10 years?
5. Both Brad and Chris purchased a \$15 000 GIC.
 - Brad's GIC has a term of 6 years and a simple interest rate of 3.2%.
 - Chris's GIC has a term of 5 years at a simple interest rate of 3.3%.
 Whose GIC will have the greater future value at maturity? Explain.
6. a) A \$12 000 Canada Savings Bond has a term of 10 years. What interest rate is needed for the future value of the CSB to be \$15 000 at maturity?
 - b) Suppose that the interest rate from part a) was increased by 1%. What would be the future value of the CSB at maturity?
7. a) Predict which investment will earn the greater amount of interest over 5 years. Explain your prediction, and then verify it.
 - A. \$1000 in a simple interest investment at 6%, paid semi-annually
 - B. \$1000 in a simple interest investment at 6%, paid monthly
 - b) Is there an advantage if interest is paid more often? Explain.
 - c) Why might someone choose investment B over investment A?
8. a) Suzette plans to invest \$10 000 and is researching the best deal online. The table below shows four investment options, with their interest rates and terms. Rank these options by their future values at maturity.



Investment Option	Simple Interest Rate	Term (years)
A	5% paid semi-annually	5
B	1.4% paid annually	10
C	1.9% paid quarterly	7
D	1.05% paid annually	8

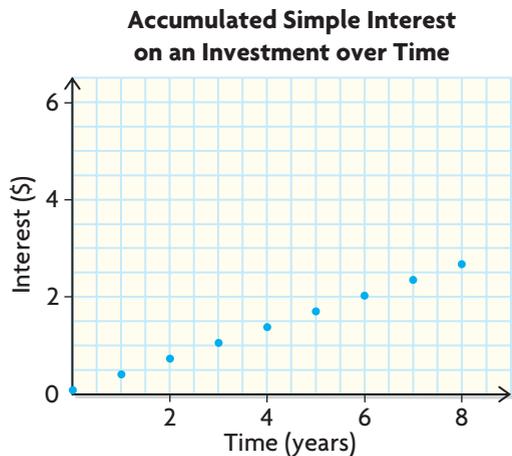
- b) Suppose that Suzette decided to withdraw her money 1.5 years before the end of the term. Would your ranking of the investment options change? Explain.

9. On July 1, Desiree deposited \$3600 into a savings account that earns 2.5% simple interest, paid daily. On the same day, her sister Latoya deposited \$3500 into a savings account that earns 3% simple interest, paid daily.
- Who will have more money on December 31? How much more?
 - Determine the difference in the interest that the sisters will earn over the 6 months.
 - Compare their rates of return.
10. Shaun has been looking at houses. He has \$10 000 that he wants to invest, hoping that he can end up with \$15 000 to make a down payment on a house. He has an opportunity to invest at 6.5% simple interest, paid annually. How long will it take before Shaun can make a down payment of \$15 000?
11. A bank is offering a simple interest rate of 3.2% for a guaranteed investment certificate with a 5-year term.
- What principal would you need to invest if you wanted to have \$20 000 at the end of the term?
 - How long would it take for the value of the GIC to be \$25 000?
12. Lin invested \$4700. After 8 years, the investment's value was \$9400.
- What was the annual simple interest rate?
 - Suppose that the interest rate continued for another 8 years. What would be the value of the investment?

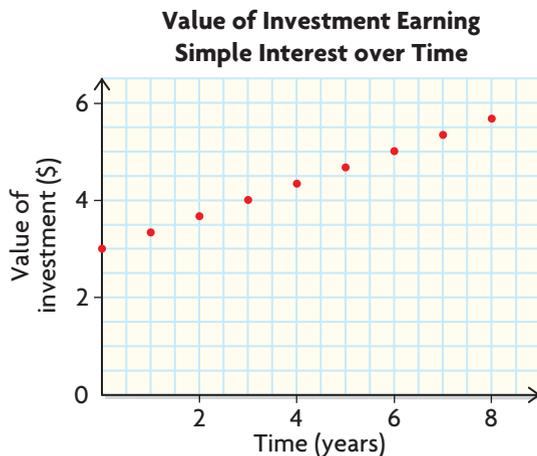


Closing

13. a) The simple interest earned on an investment represents a linear relation graphed against time. What would cause the slope of the graph to increase? Explain.



- b) The value of this investment also represents a linear relation when graphed against time. Compare the graph in part a) with the graph below. How are these situations the same and how are they different?



Extending

14. At the beginning of each year, Graham purchases a \$1000 Canada Savings Bond with a simple interest rate of 3.8%. After 5 years, what is the total value of Graham's CSBs?
15. Carole opened a savings account with \$24 000. The account earned simple interest at 5.2%, paid daily. Carole closed her account after 86 days and withdrew all her money. How much money did she withdraw?