1.4 Compound Interest: Present Value

YOU WILL NEED

- calculator
- spreadsheet software
- financial application on a graphing calculator

EXPLORE...

- Nora wants to have $5000 at the end of 3 years for college. All the interest rates that she has found are less than 10%. Determine two possible amounts that she might invest to reach her goal of $5000.

GOAL

Determine the principal or present value of an investment, given its future value and compound interest rate.

INVESTIGATE the Math

In 5 years, after graduating from college, Cal wants to spend a year travelling in Canada's three territories. He plans to start in Yukon and then travel east to the Northwest Territories and Nunavut. Cal has determined that he will need at least $15 000 for his trip. To reach this goal, he wants to invest money now. He has chosen a GIC at 7%, compounded annually.

? How much does Cal need to invest now so that he will have $15 000 in 5 years?

A. The future value of an investment that earns compound interest, given the principal or **present value**, interest rate, and compounding frequency, can be determined using the compound interest formula:

$$A = P(1 + i)^n$$

How could you use this formula to determine the present value, given the future value, interest rate, and compounding frequency?

present value

The amount that must be invested now to result in a specific future value in a certain time at a given interest rate.

B. Determine how much principal Cal needs to invest now, at 7% compounded annually, to have a future value of $15 000 in 5 years.

Reflecting

C. How could you verify your answer to part B?

D. **a)** Why might someone want to know the ratio of the future value of an investment to its present value?

b) How could you use the compound interest formula to determine this ratio?

APPLY the Math

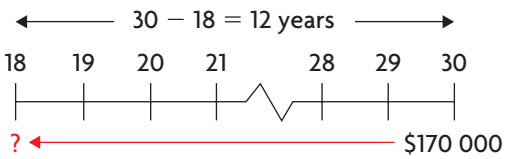
EXAMPLE 1 Determining the present value of investments earning compound interest

Ginny is 18 years old. She has inherited some money from a relative. Ginny wants to invest some of the money so that she can buy a home in Milk River, Alberta, when she turns 30. She estimates that she will need about $170 000 to buy a home.

a) How much does she have to invest now, at 6.5% compounded annually?
b) What is the ratio of future value to present value for Ginny's investment?
c) How would the ratio change if the interest rate decreased to 6% but was compounded semi-annually?

Ginny's Solution

a) interest rate is 6.5%, compounded annually



I made a timeline to record and organize the given information and visualize the problem.

I could see that I needed to know the principal, or present value, that would grow to $170 000 as it earned 6.5% interest. The term of the investment is from now until I am 30. Since I am 18, this is 12 years.

$A = 170\,000$

$i = \frac{0.065}{1}$

$n = 12$

$A = P(1 + i)^n$

$P = \frac{A}{(1 + i)^n}$

$P = \frac{170\,000}{(1.065)^{12}}$

$P = 79\,846.085...$

I need to invest \$79 846.09.

Since the compounding period is annual, the annual interest rate is also the interest rate per compounding period, and the number of years in the term is also the number of compounding periods.

I rewrote the formula for future value by isolating the present value (P), because I knew the future value (A), the interest rate per annual compounding period (i), and the number of compounding periods over the term.

b) $\frac{A}{P} = \frac{170\,000}{79\,846.09}$

$\frac{A}{P} = 2.129...$

The ratio of future value to present value for 12 years is approximately 2.13.

I determined the ratio of future value to present value, using the values from part a). I expressed the ratio as a decimal, to two decimal places.

c) $i = \frac{0.06}{2}$ or 0.03

$n = 2(12)$ or 24

$\frac{A}{P} = (1 + i)^n$

$\frac{A}{P} = (1.03)^{24}$

$\frac{A}{P} = 2.032...$

The ratio of future value to present value for 12 years is approximately 2.03. The 0.5% drop in the interest rate had more effect than doubling the compounding frequency, so the overall ratio decreased.

Instead of determining the present value again and then determining the ratio, I rewrote the compound interest formula by isolating the $\frac{A}{P}$ ratio.

Using the $\frac{A}{P}$ ratio to compare these investments makes sense since the term is the same.

Your Turn

Ginny figures that if she waited another 12 years, she would need to invest only half the present value at 6.5% compounded annually. Do you agree? Explain.

EXAMPLE 2 Determining the present value of an investment that is compounded quarterly

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that $40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a) How much money should they have invested?
b) How much interest will they earn over the term of their investment?

Gerald's Solution

a) interest rate is 9.6%, compounded quarterly

My timeline helped me visualize the problem and decide what to do.

I could see that I needed to know the present value that would grow to $40 000 as it earned 9.6% interest, compounded quarterly, for 3 years.

$A = 40\,000$

$i = \frac{0.096}{4}$ or 0.024

$n = 3(4)$ or 12

$P = \frac{A}{(1 + i)^n}$

$P = \frac{40000}{(1 + 0.024)^{12}}$

$P = 30\,092.655...$

Since the interest is compounded quarterly,
- I divided the per annum rate by 4 to determine the quarterly interest rate, and
- I multiplied the number of years in the term by 4 to determine the number of compounding periods.

I used the present value version of the compound interest formula because I knew the future value, interest rate, and number of compounding periods.

They should have invested $30 092.66.

b) $I = A - P$

$I = 40\,000 - 30\,092.66$

$I = 9907.34$

For total interest earned, I subtracted the present value from the future value.

They will earn $9907.34 in interest.

Your Turn

Would twice the present value be needed if the compounding frequency was half as often? Explain.

EXAMPLE 3 Determining an unknown interest rate and unknown term

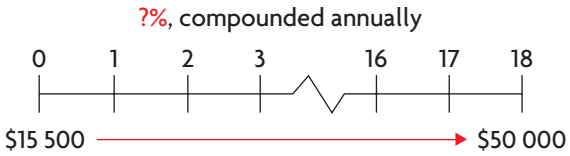
Laura has invested $15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least $50 000 by the time her newborn enters university, in 18 years.

a) What interest rate, compounded annually, will result in a future value of $50 000? Round your answer to two decimal places.

b) Suppose that Laura wants her $15 500 to grow to at least $60 000 at the interest rate from part a). How long will this take?

Frank's Solution

a)



I sketched a timeline to record and organize the known information and to visualize the problem. From my timeline, I could see that I needed to know the minimum annual interest rate, compounded annually, that is required for $15 500 to grow to $50 000 over 18 years.

$A = P(1 + i)^n$

$A = 50\,000$

$P = 15\,500$

$n = 18(1)$

$50\,000 = 15\,500(1 + i)^{18}$

$\frac{50\,000}{15\,500} = (1 + i)^{18}$

$\sqrt[18]{\frac{50\,000}{15\,500}} = 1 + i$

$1.0672... = 1 + i$

$0.0672... = i$

I substituted the known values into the compound interest formula and solved for i.

The interest rate needed to make $15 500 grow to $50 000 is 6.72%.

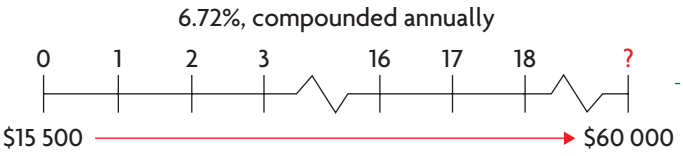
The present value is $15 500.
The annual interest rate is unknown.
The compounding frequency is annual or 1 time per year.
The term (in years) is 18.
The future value is $50 000.

I verified my answer using the financial application on my calculator.

I entered these values to determine the annual interest rate.

Interest rate = 0.067 22... or 0.0672
An interest rate of at least 6.72%, compounded annually, would give the desired future value.

b)



The present value is \$15 500.
The interest rate is 6.72%.
The compounding frequency is annual or 1 time per year.
The term (in years) is unknown.
The future value is \$60 000.

Term = 20.810...

It will take 21 years for \$15 500 to grow to at least \$60 000 at the interest rate from part a), since compounding is annual.

I revised my timeline.

I knew that more than 18 years would be needed for \$15 500 to grow to \$60 000, since \$15 500 grew to \$50 000 in 18 years at the same interest rate.

I had planned to use the compound interest formula, but realized that I couldn't solve this equation for n: $\frac{A}{P} = (1 + i)^n$

I used my financial application to avoid solving for an unknown that is an exponent. I entered these values to determine the term.

No interest will be paid on the last 0.810 year.

Your Turn

Could Frank have solved either of these problems if he had not known the compounding frequency that was expected? Explain.

In Summary

Key Idea

- The present value of an investment that earns compound interest can be determined using the formula

$$P = \frac{A}{(1 + i)^n}$$

where P is the present value (or principal), A is the amount (or future value), i is the interest rate per compounding period (expressed as a decimal), and n is the number of compounding periods.

Need to Know

- Any equivalent form of the compound interest formula may be used to solve a compound interest problem.

$A = P(1 + i)^n$ $\quad P = \frac{A}{(1 + i)^n}$ $\quad \frac{A}{P} = (1 + i)^n$

- To compare investments, usually with the same term or principal, the ratio of the future value to the present value can be determined using the form: $\frac{A}{P} = (1 + i)^n$
- Using a formula, using the financial application on a graphing calculator, and using spreadsheet software are all valid strategies for solving a compound interest problem.

CHECK Your Understanding

1. Predict which investment will require a greater present value to be invested. Explain your prediction, and then verify it.

A. Future value of $10 000 at 5%, compounded monthly, for 10 years

B. Future value of $10 000 at 5%, compounded quarterly, for 10 years

2. **a)** Determine the future value to present value ratio for both investments in question 1.

b) Would an investment with a future value of $10 000 at 6%, compounded annually, for 10 years have a higher or lower ratio? Explain.

3. Complete the table.

Future Value (amount in $)	Present Value (principal in $)	Interest Rate per Annum (%)	Compounding Frequency	Investment Term (years)
2 500.00	?	7.8	annually	8
3 500.00	2000.00	?	semi-annually	5
11 000.00	?	2.4	quarterly	12
100 000.00	609.35	13.6	annually	?
23 500.00	16 150.00	?	monthly	2

4. Mac plans to retire in 20 years, when he is 55. He estimates that he will need $250 000 to live on, until he is eligible for his pension.

a) How much money should he invest now, at 8.5% compounded annually, to meet his goal?

b) How much interest will he earn in 20 years?

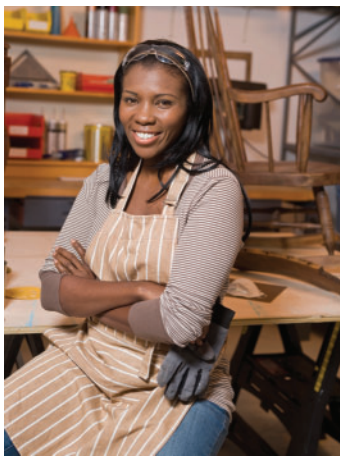
PRACTISING

5. Joseppie is planning to buy a new snowmobile in 2 years. He intends to spend no more than $17 000. He has $9000 to invest in an account that compounds interest quarterly.

a) What rate of interest will Joseppie need to find in order to meet his goal? Is his plan reasonable?

b) Suppose that he manages to find an interest rate of 12%, compounded quarterly. How long will it take him to save enough money?

6. Claire wants a down payment of $17 500 to buy a house in 10 years, when she turns 30. Her bank offers her an investment with 5.6% interest, compounded semi-annually. What present value will she need to invest now?



7. Sasha predicts that she will need $24 000 to remodel her carpentry workshop in 6 years. She has found three investment options to consider:

A. 4.80%, compounded annually
B. 4.75%, compounded semi-annually
C. 4.70%, compounded quarterly

a) Compare the rates of return for these three options. Which option should she choose? Why?
b) How much interest will she earn?

8. Choose one of these investments:

A. $15 000 GIC that earns 3.8%, compounded annually, for 10 years
B. $26 000 CSB that earns 6.2% for the first 5 years, compounded semi-annually
C. $8000 investment savings account that earns 4.1%, compounded quarterly, for 8 years

a) Graph the value of the investment ($) against time (years).
b) Change either the interest rate or the principal. Graph the value of the new investment against time on the same grid.
c) Make another change to the same variable you changed in part b). Graph the value of the new investment against time on the same grid.
d) How did the changes in the variable affect the shape of the graph?

9. In 40 years, Blake wants to have $1 000 000. He plans to invest less than $10 000 now. Which of these investment options would allow him to invest the least and still meet his goal? Justify your choice.

A. 12.6%, compounded annually
B. 11.9%, compounded semi-annually
C. 13.2%, compounded quarterly
D. 11.53%, compounded weekly

10. Franco invested money at 6.9%, compounded annually, while David invested money at 6.9%, compounded monthly. After 30 years, each investment is worth $25 000. Who made the greater original investment, and by how much was it greater?

11. Lucy is investing $3000. She wants it to grow to $7000 in 10 years.

a) What annual rate of interest, compounded quarterly, does Lucy need to meet her goal? Round your answer to two decimal places.
b) What is the ratio of future value to present value for Lucy's investment? Predict whether this ratio would increase or decrease if Lucy invested $3000 at the same interest rate, but compounded annually. Explain your prediction, and verify it.

12. Daniel has a savings account that earns interest at 5.3%, compounded monthly. He has not made any deposits or withdrawals in the past 9 months. If there is $4765.30 in the account today, how much interest has the account earned in the past 9 months?

13. Ben would like to send his parents on a $15 000 safari for their 35th wedding anniversary in 10 years. He has the opportunity to invest in a GIC that earns 5.5%, compounded semi-annually. His brother and sister have agreed to split the cost of the GIC with him. How much will each sibling contribute to the cost of the GIC?

Closing

14. Imagine that you are a financial advisor. You have a client who knows nothing about investments. Explain the key features that your client should look for in a fixed-interest investment opportunity. Use the terms *present value*, *future value*, *simple interest*, *compound interest*, *interest rate*, *compounding frequency*, and *term* in your explanation.

Extending

15. What annual interest rate, compounded quarterly, would enable an investment to triple every 12 years? Round your answer to two decimal places.

16. **a)** What is the future value of $1000 if invested for 1 year at 5%, compounded annually?

b) What interest rate would result in the future value from part a) for each compounding frequency?

i) semi-annually **ii)** quarterly **iii)** monthly

c) What advantage might there be in choosing a lower interest rate that is compounded more frequently?