1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: Why would someone choose an investment that paid compound interest over an investment that paid simple interest, assuming that the principal, interest rate, and term are the same?

A: Simple interest is earned only on the principal of the investment, while compound interest is earned on the principal and any accumulated interest. So, when the principal, interest rate, and term are the same, a compound interest investment will earn more interest than a simple interest investment. For example, consider both investments below.

Sample Simple Interest Investment
P = \$2000, r = 8%, t = 3 years

Term (year)	Value at Start of Year ($)	Interest Rate	Interest Earned ($)	Value at End of Year ($)
1	2000	0.08	160	2160
2	2160	0.08	160	2320
3	2320	0.08	160	**2480**

Sample Annual Compound Interest Investment
P = \$2000, i = 8%, n = 3

Term (year)	Value at Start of Year ($)	Interest Rate	Interest Earned ($)	Value at End of Year ($)
1	2000.00	0.08	160.00	2160.00
2	2160.00	0.08	172.80	2332.80
3	2332.80	0.08	186.62	**2519.42**

The compound interest investment earns \$39.42 more interest than the simple interest investment after 3 years.

Study *Aid*
- See Lessons 1.1 and 1.2.
- Try Mid-Chapter Review Questions 1 to 3.

Study Aid

- See Lessons 1.3 and 1.4.
- Try Mid-Chapter Review Questions 4 to 9.

Q: How do you determine what values to substitute for the variables *n* and *i* when using the compound interest formula,

$$A = P(1 + i)^n$$

A: The number of compounding periods (n) is determined by using the compounding frequency and the term (t). The interest rate per compounding period (i) is the quotient of the annual interest rate (r) and the compounding frequency. Use an exact value for i so there is no rounding error. For example, when \$1000 is invested at 5% for 10 years:

Semi-annual compounding:

$i = \frac{r}{2}$ $\qquad n = t(2)$

$i = \frac{0.05}{2}$ $\qquad n = 10(2)$

$i = \mathbf{0.025}$ $\qquad n = \mathbf{20}$

$A = P(1 + i)^n$

$A = 1000(1 + \mathbf{0.025})^{\mathbf{20}}$

Monthly compounding:

$i = \frac{r}{12}$ $\qquad n = t(12)$

$i = \frac{\mathbf{0.05}}{\mathbf{12}}$ $\qquad n = 10(12)$

$n = \mathbf{120}$

$A = P(1 + i)^n$

$A = 1000\left(1 + \frac{\mathbf{0.05}}{\mathbf{12}}\right)^{\mathbf{120}}$

Study Aid

- See Lessons 1.3 and 1.4.
- Try Mid-Chapter Review Questions 4 to 10.

Q: How do you know what form of the compound interest formula to use when solving a problem?

A: You can use any equivalent form of the compound interest formula, but you might prefer to use a form that simplifies your calculations. For example, when \$1000 is invested at 5%, compounded semi-annually, for 10 years, it grows to \$1638.62:

Determining the present value, when future value, annual interest rate, compounding frequency, and term are known:

$P = \frac{A}{(1 + i)^n}$

$P = \frac{1638.62}{(1 + \frac{0.05}{2})^{2(10)}}$

$P = \frac{1638.62}{(1 + \frac{0.05}{2})^{20}}$

$P = \$1000$

Determining the annual interest rate, when future value, present value, compounding frequency, and term are known:

$\frac{A}{P} = (1 + i)^n$

$\frac{1638.62}{1000} = (1 + i)^{20}$

$1.638\,62^{\frac{1}{20}} = [(1 + i)^{20}]^{\frac{1}{20}}$

$1.025\,000\,111... = 1 + i$

$0.025\,000\,111... = i$

$r = 2(0.025\,000\,111...)$

$r = 0.050...$ or 5%

You can also use a financial application to determine any unknown variable in a compound interest problem situation if you know the other variables. This is recommended when determining the term of the investment.

PRACTISING

Lesson 1.1

1. Paula earned $27.54 in simple interest by investing $450. The interest rate was 2.04%. For how many years did she hold the investment?

2. **a)** For how long would $6000 need to be invested, at 6.4% simple interest, to earn $1200 in interest?

b) How long would it take if the interest for part a) was paid yearly?

c) How long would it take if the interest was paid quarterly?

Lesson 1.2

3. Brad and Katherine deposited $5000 in two separate accounts when their baby was born. Katherine's account earns compound interest at 4.87%, paid annually. Brad's account earns simple interest at 5.5%. The investments will mature when their child turns 20.

a) What will each account be worth at maturity?

b) Graph both investments on the same coordinate grid.

c) What does the intersection point of the two graphs represent?

Lesson 1.3

4. Ron purchased a 10-year GIC for $3000. The GIC earns 5.6% interest, compounded annually.

a) What will be the future value of the GIC at maturity?

b) Estimate how long it will take for the GIC to be worth at least $12 000.

c) Predict what would happen to the future value of the GIC in each situation below. Explain your prediction, and then verify it.

i) The compounding frequency is monthly.

ii) The interest rate is 2.8%, compounded semi-annually.

d) What minimum interest rate, with daily compounding, would be needed to have a future value that is $100 greater than the future value you determined in part a)?

5. An alumnus of a local high school donated $50 000 to the school. The amount was invested for 3 years at 7.75%, compounded quarterly. The school has agreed to use only the interest earned on the investment to buy sports equipment. How much money will be available for sports equipment at the end of the investment's term?

6. Rank the following changes by their impact on the future value of $1000 invested at 5%, compounded annually, for 5 years:

A. Increasing the principal to $1050

B. Increasing the interest rate to 6%

C. Increasing the compounding frequency to monthly

D. Increasing the term to 6 years

Lesson 1.4

7. **a)** How much should Desiree invest at 6%, compounded monthly, to have $10 000 in 3 years?

b) How much should Desiree invest if the compounding period is semi-annual?

8. On Petra's 22nd birthday, she received a gift of $11 000. This was the future value of an investment that was made when she was born.

a) How much was invested 22 years ago

i) if the interest rate was 7.2%, compounded annually?

ii) if the interest rate was 7.2%, compounded semi-annually?

b) Suppose that $11 000 had been invested at 7.2% interest, compounded monthly, for 22 years. Would the ratio of future value to present value be higher or lower than the ratio for the original investment? Justify your answer.

9. An investment of $400 grew to $625 in 10 years. What was the annual interest rate if the interest was compounded monthly?

10. An investment of $250 grew to $1000 at 6% interest, compounded semi-annually. Estimate how long it took for the investment to grow, and then verify your estimate.