FREQUENTLY ASKED Questions

Q: How can you represent the elements in a set?
A: If the set is finite, you can do the following:
- Describe the set. For example, 
  \[ E = \{\text{even numbers between 2 and 10}\}. \]
- List the elements in any order. For example, the set of days that begin with T can be represented as 
  \[ T = \{\text{Thursday, Tuesday}\}. \]
- Use set notation. For example, the set of positive integers less than 1000 can be represented as follows:
  \[ P = \{x \mid 0 < x < 1000, \ x \in \mathbb{I}\} \]

If a set is infinite, you can either describe it or use set notation. For example:
- \[ L = \{\text{the set of rational numbers greater than or equal to 25}\}, \]
- \[ L = \{k \mid k \geq 25, \ k \in \mathbb{Q}\} \]

You can choose any letter to name a set.

Q: How does a Venn diagram show how sets are related?
A: For example, in the Venn diagram below, sets \( A \) and \( B \) overlap because they share common elements. Set \( C \) is inside set \( B \) because it is a subset of set \( B \). Sets \( A \) and \( C \) do not overlap because they do not share any elements; that is, they are disjoint. The complement of a set, or sets, consists of all the elements of the universal set that are not in the original set. A Venn diagram shows the relationships between sets but not their relative sizes.
Q: What is meant by the intersection and union of two sets?

A: The intersection of two sets, \( A \) and \( B \), consists of the elements that are common to both sets. It is represented as \( A \cap B \) and is read as “\( A \) and \( B \)”.

If there are no common elements, the intersection is the empty set.

The union of two sets is the elements in the first set only, the second set only, and the intersection of both sets. It is represented as \( A \cup B \) and read as “\( A \) or \( B \)”.

For example, consider the following two sets of playing cards:

\[ H = \{A\heartsuit, 3\heartsuit, 5\heartsuit, 7\heartsuit, 9\heartsuit, Q\heartsuit, K\heartsuit\} \]
\[ F = \{J\clubsuit, Q\clubsuit, K\clubsuit, J\diamondsuit, Q\diamondsuit, K\diamondsuit, J\spadesuit, Q\spadesuit, K\spadesuit\} \]

The intersection:
\[ H \cap F = \{J\heartsuit, Q\heartsuit, K\heartsuit\} \]

The union:
\[ H \cup F = \{A\heartsuit, 3\heartsuit, 5\heartsuit, 7\heartsuit, 9\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit, J\diamondsuit, Q\diamondsuit, K\diamondsuit, J\spadesuit, Q\spadesuit, K\spadesuit\} \]

Q: How can you determine the number of elements in the union of two sets?

A: Use the Principle of Inclusion and Exclusion:

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]

If the sets are disjoint, the intersection is the empty set. For example, consider the sets in the previous example:

\[ n(H) = 8, \quad n(F) = 12, \quad \text{and} \quad n(H \cap F) = 3 \]

\[ n(H \cup F) = 8 + 12 - 3 = 17 \]
**PRACTISING**

**Lesson 3.1**

1. Lucy drew the following Venn diagram:

   ![Venn Diagram]

   a) Using set notation and sets V, M, F, and N, list the subsets.
   b) How might Lucy have chosen what to put in each set?
   c) Is $M'$ equal to V? Explain.
   d) List the disjoint sets, if there are any.

2. a) Draw a Venn diagram to show:
   - the universal set $U = \{\text{natural numbers from 1 to 40}\}$
   - $E = \{\text{multiples of 3}\}$
   - $F = \{\text{multiples of 15}\}$
   - $S = \{\text{multiples of 9}\}$
   b) List the disjoint subsets, if there are any.
   c) Is each statement true or false? Explain.
      i) $E \subset F$
      ii) $S \subset E$
      iii) $S \subset F$
      iv) $F' = \{\text{all numbers from 1 to 40 except 15 and 30}\}$
      v) $S \subset S$

3. Make a list of different sports equipment. Organize the equipment into sets using a Venn diagram.

**Lesson 3.2**

4. Jordan asked 40 students at his school cafeteria what they bought for lunch. He recorded his results in the table below.

<table>
<thead>
<tr>
<th>Purchase</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>beverage</td>
<td>34</td>
</tr>
<tr>
<td>soup</td>
<td>18</td>
</tr>
<tr>
<td>no beverage or soup</td>
<td>5</td>
</tr>
</tbody>
</table>

**Lesson 3.3**

5. A total of 20 students are on a field trip. Of these students, 13 are wearing sunglasses and 6 are wearing a hat. Only 5 students are not wearing sunglasses or a hat.
   a) How many students are wearing both sunglasses and a hat?
   b) How many students are wearing sunglasses but not a hat?
   c) How many students are wearing a hat but not sunglasses?

6. Tanya was given the following sets and asked to represent them using a Venn diagram:
   - $U$ is the universal set.
   - $A$ and $B$ are subsets of $U$.
   - $n(U) = 40$, $n(A) = 16$, and $n(B) = 19$
   - $n(A \cup B') = 8$

   She drew the following Venn diagram:

   ![Venn Diagram]

   a) Tanya made an error in her Venn diagram. What was her error? Explain.
   b) Redraw Tanya’s Venn diagram correctly.

7. Paul asked 20 students whether they have a dog or a cat.
   - 4 students do not have a dog or a cat.
   - 8 students have a dog.
   - 8 students have a cat.

   How many students have both a dog and a cat?