## The Tower of Hanoi

The Tower of Hanoi is a puzzle that consists of three vertical rods and a number of disks of different sizes, which can be moved and placed onto any rod. The puzzle starts with all the disks arranged in an orderly stack as shown—the smallest at the top and the largest at the bottom. The goal is to move the entire stack of disks to another rod, following these rules:

- Only one disk may be moved at a time.
- Each move involves taking the upper disk from one of the rods and placing it onto another rod, on top of any other disks that may already be present on that rod.
- No disk may be placed on top of a smaller disk.

The Tower of Hanoi puzzle is called a counting problem. Counting problems involve situations where you determine all the possibilities.
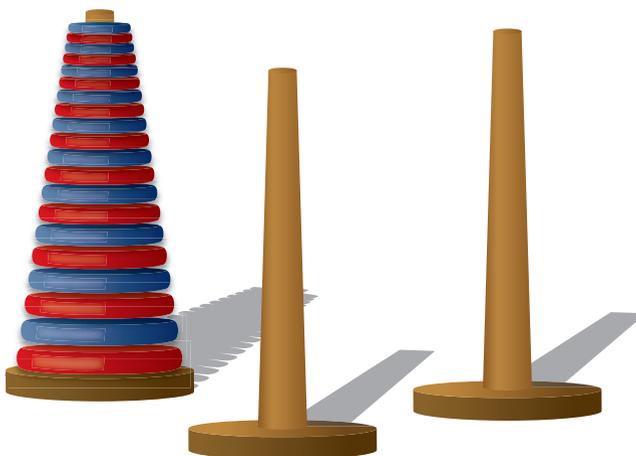
? How can you determine the minimum number of moves needed to complete the puzzle with a stack of 20 disks, without counting every move?

A. Suppose you started with a stack consisting of 1 disk. How many moves will it take to move the stack to another rod? What if you started with a stack of 2 disks?

B. Try the puzzle, starting with stacks of 1, 2, 3, 4, and then 5 disks. Count and record the minimum number of moves in a table like this:

Number of Disks in Stack	1	2	3	4	5
Minimum Number of Moves					

C. Compare your completed table with your classmates' tables. Discuss how the disks must be moved so the minimum number of moves are used in each case.

D. Describe the pattern in your completed table.

E. Use the pattern to predict the minimum number of moves for stacks of 6 disks to 10 disks, and extend your table to record your predictions.

F. What type of reasoning did you use to analyze this puzzle: **deductive reasoning** or **inductive reasoning**? How do you know?

G. Examine your table from part E. How do the numbers in the Minimum Number of Moves row relate to powers of 2? Write an algebraic expression to represent the minimum number of moves needed for a stack of n disks.

H. Use your expression from part G to determine the minimum number of moves for a stack of 20 disks.

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. The best way to solve a counting problem is to list all the possibilities and then count them. For example, to determine all the different ways a handful of coins can land when flipped all at the same time, list every possible combination of heads and tails and count them.

2. Looking for and extending a pattern is a useful strategy for solving counting problems.

3. There are three people in a group: Sam, Ted, and Jill. Consider these situations:

- Three people in a line
- Three people who can wash cars at a fundraiser

When counting the number of ways each situation could occur, Jim claims that the order of the people in the group matters in only one of the situations.