4 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study Aid
- See Lesson 4.1, Examples 1 to 3.
- Try Mid-Chapter Review Questions 1 to 4.

Q: What is the Fundamental Counting Principle and what is it used for?

A: The Fundamental Counting Principle is used to calculate the number of ways a set of tasks can occur in a counting problem. The product of the number of ways each task can occur results in the number of ways that all the tasks can occur. For example:

> A breakfast menu offers four choices of eggs (sunny side up, scrambled, poached, soft boiled), two choices of toast (white, whole wheat), and three choices of meat (bacon, sausage, ham). If you must order one item from each category, the number of possible breakfasts, B, is
>
> $$B = 4 \cdot 2 \cdot 3, \text{ or } 24$$

Look for the connecting word AND between each task to identify when the Fundamental Counting Principle should be used.
If a problem involves two or more tasks connected by the word OR, the Fundamental Counting Principle does not apply. In this type of problem, the number of ways each task can occur are added together.

Study Aid
- See Lesson 4.2, Examples 1 to 4.
- Try Mid-Chapter Review Questions 5 to 8.

Q: What is factorial notation and what does $n!$ represent?

A: Factorial notation is a short form for representing the product of decreasing consecutive natural numbers.

$$n! = (n)(n - 1)(n - 2)...(3)(2)(1)$$

The expression $n!$ can be used to calculate the number of ways you can make ordered arrangements, or permutations, using n different objects. For example, the number of arrangements of five people in a line, A, is

$$A = 5!$$
$$A = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1, \text{ or } 120$$

Study Aid
- See Lesson 4.3, Examples 1 to 5.
- Try Mid-Chapter Review Questions 9 to 12.

Q: How do you determine the number of permutations possible from a group of different objects when you do not use all of them in each permutation?

A: Suppose you have a group of n different objects, and you need to create ordered arrangements using only r of those objects. You can calculate the number of ways this can be done using the formula

$$_nP_r = \frac{n!}{(n - r)!}$$

For example, the number of ways that a president and vice-president can be selected from a group of 10 candidates is

$$_{10}P_2 = \frac{10!}{(10 - 2)!}$$

PRACTISING

Lesson 4.1

1. A sub shop offers the following choices:
- 3 types of buns
- 5 types of cold cuts
- 3 types of cheese
- 12 different toppings
- 3 different sauces

If Mario wants a sub with one item from each category, how many different subs can he choose from?

2. Radio and television stations in the USA and Canada have four-character station names. There are 3 choices for the first letter: K, W, and C, and a blank may be used as the last character. For instance, both CKLW and WHO are acceptable names. How many station names are possible?

3. How many ways can a sum of 2 or a sum of 10 be rolled with a pair of standard dice?

4. How many ways can you select 3 horses to come first, second, and third in a 10-horse race?

Lesson 4.2

5. Evaluate each expression.

a) $8!$ **b)** $6! \cdot 3!$ **c)** $\frac{9!}{6!}$ **d)** $\frac{10 \cdot 9!}{5 \cdot 8!}$

6. How many different lineups can be formed by nine players on a softball team?

7. Simplify each expression.

a) $(n + 5)(n + 4)!$ **c)** $\frac{(n - 4)!}{(n - 5)!}$

b) $\frac{(n + 4)!}{(n + 2)!}$ **d)** $\frac{(n + 2)!}{n!}$

8. Solve each equation.

a) $\frac{n!}{(n - 2)!} = 72$ **b)** $\frac{(n - 1)!}{(n - 3)!} = 30$

Lesson 4.3

9. Evaluate the following expressions.

a) ${}_9P_2$ **b)** ${}_{12}P_8$ **c)** ${}_5P_5$ **d)** ${}_{12}P_{10}$

10. State the restrictions on the variable, n, for each expression in

a) question 7

b) question 8

11. Rennie has 20 CDs in his car. His CD player holds 6 CDs. How many different ways can he load his CD player?

12. Manny is the captain of the 15-member soccer team that has won his city's championship. How many ways can Manny and 2 other players line up to receive the championship trophy, if the captain must be first in line?

13. Margo claims that for some counting problems, you can use either the Fundamental Counting Principle or the permutations formula,

$$ {}_nP_r = \frac{n!}{(n - r)!} $$

Do you agree? Use examples to support your position.