4.5 Exploring Combinations

GOAL

Explore how counting combinations differs from counting permutations.

YOU WILL NEED

- calculator

EXPLORE the Math

The school athletic council has five members: Jill, Ted, Rhaj, Yvette, and Martin. This Wednesday, they plan to hold a bake sale.

? How can you count the number of committees of at least two people that can be chosen to sell baked goods during lunch?

Reflecting

A. Share your results and your strategy with several other pairs of students. Discuss any differences.

B. When you considered all the possible committees, was each committee a permutation or a **combination** of people? Explain.

C. The expression $\frac{5!}{a!b!}$ can be used to determine the number of committees that can be formed from 5 people. What do a and b represent in this expression? Verify that this expression works for all the 2-, 3-, 4- and 5-person committees you considered.

D. Suppose only 1 person is needed for the bake sale. How many ways are there to choose that person? Does the expression in part C work in this situation?

E. What is the relationship between the number of people to choose from, in this case 5 people, and the variables a and b in the expression $\frac{5!}{a!b!}$?

combination

A grouping of objects where order does not matter. For example, the two objects *a* and *b* have one combination because *ab* is the same as *ba*.

In Summary

Key Ideas

- When order does not matter in a counting problem, you are determining combinations. For example, *abc*, *acb*, *bac*, *bca*, *cab*, and *cba* are the six different permutations of the letters *a*, *b*, and *c*, but they all represent the same single combination of letters.
- When all n objects are being used in each combination, there is only one possible combination.

Need to Know

- From a set of n different objects, there are always fewer combinations than permutations when selecting r of these objects where $r \leq n$. For example, the number of permutations, P, possible using two of the three letters *a*, *b*, and *c* is

$$P = \frac{3!}{(3-2)!}$$
$$P = 6$$

The six permutations are *ab*, *ba*, *ac*, *ca*, *bc*, and *cb*. However, of these six permutations, only *ab*, *ac*, and *bc* are different two-letter combinations.

FURTHER Your Understanding

1. Brian, Rachelle, and Linh volunteer at the food bank on Saturdays.

a) In the morning, each person is needed for a different job: stacking cans, stocking dry goods, and cleaning fruits and vegetables. In how many different ways can they be chosen for these jobs?

b) List all the ways the three volunteers can be assigned the jobs in part a).

c) In the afternoon, all three volunteers are needed to unload vehicles arriving from food drives. In how many ways can they be chosen for this job?

d) In the situations above, which involved permutations and which involved combinations? Explain how you know.

2. Explain the main difference between the following:

- Permutations of 4 objects out of a group of 6 different objects
- Combinations of 4 objects out of a group of 6 different objects

3. There are 10 members of student council. How many ways can 4 of the members be chosen to serve on the dance committee?

4. There are 12 dogs at the local animal shelter. To increase the likelihood that the dogs will be adopted, 3 of them will appear on a TV morning show. How many ways can 3 of the 12 dogs be selected to appear?