5.2 Probability and Odds

GOAL

Understand and interpret odds, and relate them to probability.

EXPLORE...

- An oil and vinegar salad dressing is made using 2 parts oil to 1 part vinegar. So, the ratio of oil : vinegar is 2 : 1. What fraction of the dressing is oil?

INVESTIGATE the Math

Suppose that, at the beginning of a regular CFL season, the Saskatchewan Roughriders are given a 25% chance of winning the Grey Cup.

? What are the odds of the Roughriders winning the Grey Cup?

A. What is the event in the situation described above?

B. Express the probability that this event will occur as a fraction out of 100.

C. Describe the **complement** of this event.

D. Express the probability that the complement will occur as a fraction out of 100.

E. Write the **odds in favour** of the Roughriders winning the Grey Cup.

F. Write the **odds against** the Roughriders winning the Grey Cup.

odds in favour

The ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes.

odds against

The ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes.

Reflecting

G. How are the odds in favour of the Roughriders winning related to the odds against them winning?

H. How does the probability of an event and the probability of its complement relate to the odds in favour of the event occurring?

I. How are the odds in favour of an event similar to its probability? How are they different from its probability?

APPLY the Math

EXAMPLE 1 Determining odds using sets

Bailey holds all the hearts from a standard deck of 52 playing cards. He asks Morgan to choose a single card without looking.

Determine the odds in favour of Morgan choosing a face card.

Morgan's Solution

Let H be the universal set of all hearts:
$H = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$

I determined and listed the elements in the universal set H.

Let C be the set of face cards that are hearts:
$C = \{J♥, Q♥, K♥\}$

I determined and listed the elements in the subset C. This is the subset that defines the event I was interested in: drawing a face card.

Let C' be the set of cards in the suit of hearts that are not face cards.
$C' = \{A♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥\}$

Set C is all the face cards that are hearts, which are favourable outcomes. The other cards are in the universal set, but not in set C. They make up the complement of C, or C'. Thus, C' is the set of unfavourable outcomes.

Odds in favour $= n(C) : n(C')$
Odds in favour $= 3:10$

The odds in favour of choosing a face card are 3:10.

To determine the odds in favour of the event, I counted the number of elements in sets C and C'. Then I formed the ratio of favourable outcomes to unfavourable outcomes.

Your Turn

a) Determine the odds against Morgan drawing a face card.
b) Compare the odds in favour of this event to the odds against it. Do you think Morgan is more likely to draw a face card or something different? Explain.

EXAMPLE 2 Determining odds from probability

Research shows that the probability of an expectant mother, selected at random, having twins is $\frac{1}{32}$.

a) What are the odds in favour of an expectant mother having twins?
b) What are the odds against an expectant mother having twins?

Lesley's Solution

a) $P(\text{twins}) = \frac{1}{32}$

So, the ratio of twins to all birth combinations is $1:32$.

I interpreted the probability of having twins. I started by writing this probability as a part-to-whole ratio.

$P(\text{not twins}) = \frac{31}{32}$

So, the ratio of birth combinations that are not twins to all birth combinations is $31:32$.

I determined the complement of P(twins) and wrote it as a part-to-whole ratio.

The odds in favour of having twins, $P(\text{twins}) : P(\text{not twins})$, are $1:31$.

I wrote the odds in favour of having twins as a part-to-part ratio. The first term is the number of favourable outcomes from the probability of having twins. The second term is the number of unfavourable outcomes.

b) The odds against having twins, $P(\text{not twins}) : P(\text{twins})$, are $31:1$.

To determine the odds against having twins, I simply switched the terms in the odds in favour of having twins.

Your Turn

Suppose that the probability of an event happening is $\frac{2}{5}$.

a) What are the odds in favour of the event happening?
b) What are the odds against the event happening?

EXAMPLE 3 Determining probability from odds

A computer randomly selects a university student's name from the university database to award a \$100 gift certificate for the bookstore. The odds against the selected student being male are $57:43$. Determine the probability that the randomly selected university student will be male.

Cheyenne's Solution

Total number of outcomes $= 57 + 43$
Total number of outcomes $= 100$

I determined a number to represent the total number of possible outcomes by adding the terms for unfavourable and favourable outcomes in the odds against ratio.

For every 100 students, 57 are female and 43 are male.

$P(\text{male}) = \dfrac{43}{100}$

The probability that a randomly selected university student will be male is 43%.

I expressed the probability as a ratio of the number of favourable outcomes to the total number of possible outcomes.

Your Turn

Suppose that the odds in favour of an event are 5 : 3. Is the probability that the event will happen greater or less than 50%?

EXAMPLE 4 Making a decision based on odds and probability

A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records.

Player	Attempts	Goals Scored
Ellen	13	8
Brittany	17	10

Who should go first?

Coach's Solution

Ellen has 13 attempts and has scored 8 goals. This means that she has $13 - 8$ or 5 attempts where she did not score. The odds in favour of her scoring are 8 : 5.
Brittany has 17 attempts and scored 10 goals, so the odds in favour of her scoring are 10 : 7.

I wrote the odds in favour of each player scoring in a shootout.

Brittany's record looked better, but I was not positive. It was tough to compare the odds in favour, since the ratios involve different numbers.

The probability that Ellen will score is $\frac{8}{13}$ or about 0.615.

The probability that Brittany will score is $\frac{10}{17}$ or about 0.588.

Since $0.615 > 0.588$, there is a better chance that Ellen will score. Therefore, Ellen should go first.

I converted the players' probabilities of scoring to decimals, so they would be easier to compare.

Ellen is more likely to score, based on her shootout record.

Your Turn

Josie is also on the team. She has scored twice in three shootout attempts. Should the coach put her first? Explain.

EXAMPLE 5 Interpreting odds against and making a decision

A group of Grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5 : 2, and the odds against winning Zap are 7 : 3. Which game should Madison play?

Madison's Solution: Changing odds to probability

The odds against winning Bim are 5 : 2.
The total number of outcomes is 5 + 2 or 7.
So, if I play Bim 7 times, I am likely to lose 5 times and win 2 times.

I determined the probability that I would win Bim.

$$P(\text{winning Bim}) = \frac{\text{number of wins}}{\text{number of games}}$$

$$P(\text{winning Bim}) = \frac{2}{7}$$

The odds against winning Zap are 7 : 3.
The total number of outcomes is 7 + 3 or 10.
So, if I play Zap 10 times, I am likely to lose 7 times and win 3 times.

I used the same method to determine the probability that I would win Zap.

$$P(\text{winning Zap}) = \frac{3}{10}$$

$P(\text{winning Bim}) = 0.285...$
$P(\text{winning Zap}) = 0.3$

I wrote the fractions as decimals so that I could compare them. It is easier to see that 0.3 is greater than 0.285... than it is to see that $\frac{3}{10}$ is greater than $\frac{2}{7}$.

I should play Zap, since I am more likely to win.

Samara's Solution: Using equivalent ratios

Odds against winning Bim:

$5:2$

$(5 \times 3):(2 \times 3)$

$15:6$

The odds against winning Bim are $15:6$.

Odds against winning Zap:

$7:3$

$(7 \times 2):(3 \times 2)$

$14:6$

The odds against winning Zap are $14:6$.

I knew that if I wrote the odds ratios as equivalent ratios with the same second term, I would be able to compare the first terms directly.

I multiplied each term in the ratio $5:2$ by 3 and each term in the ratio $7:3$ by 2. The result was two ratios with a second term of 6.

Since $15 > 14$, the odds against winning Bim are greater. Therefore, Madison should play Zap.

The odds against winning Bim are greater than the odds against winning Zap.

Your Turn

The Grade 12 students want to include one more game at their charity carnival. They need to choose between game A and game B. The odds against winning game A are $11:3$, and the odds against winning game B are $17:6$. The goal is to raise as much money as possible for the animal shelter. Which game should the students choose? Assume that people are equally likely to play the two games.

In Summary

Key Ideas

- Odds express a level of confidence about the occurrence of an event.
- The odds in favour of event A occurring are given by the ratio $\frac{P(A)}{P(A')}$ or $P(A):P(A')$.
  This simplifies to the ratio of favourable outcomes to unfavourable outcomes.
- The odds against event A occurring are given by the ratio $\frac{P(A')}{P(A)}$ or $P(A'):P(A)$.
  This simplifies to the ratio of unfavourable outcomes to favourable outcomes.

Need to Know

- $P(A')$ is the probability of the complement of A, where $P(A') = 1 - P(A)$.
- If the odds in favour of event A occurring are $m:n$, then the odds against event A occurring are $n:m$.
- If the odds in favour of event A occurring are $m:n$, then $P(A) = \frac{m}{m + n}$.

CHECK *Your Understanding*

1. The odds in favour of Marcia passing her driver's test on the first try are 5 : 3.
   a) Determine the odds against Marcia passing her driver's test.
   b) Determine the probability that she will pass her driver's test.

2. Colby has 10 coins in his pocket, and 3 of these coins are loonies. He reaches into his pocket and pulls out a coin at random.
   a) Determine the probability of the coin being a loonie.
   b) Determine the odds against the coin being a loonie.

3. Lily draws a card at random from a standard deck of 52 playing cards.
   a) Determine the probability of the card being red.
   b) Determine the odds in favour of the card being red.
   c) Determine the odds against the card being a spade.
   d) Determine the probability of the card being a face card.

PRACTISING

4. Mina notices that apple juice is on sale at a local grocery store. The last five times that apple juice was on sale, it was available only twice.
   a) Determine the odds in favour of apple juice being available this time.
   b) Determine the odds against apple juice being available this time.

5. There are 30 students in Mario's Grade 12 math class. The odds in favour of two students sharing a birthday are 7 : 3. Determine the probability of two students sharing a birthday.

6. Jamia likes to go wall climbing with her friends. In the past, Jamia has climbed to the top of the wall 12 times in 24 attempts.
   a) Determine the probability of Jamia climbing to the top this time.
   b) Determine the odds against Jamia climbing to the top.
   c) The odds from part b) are called "even odds." Explain what this term might mean.

7. The weather forecaster says that there is a 60% probability of snow tomorrow. What are the odds against snow?

8. About 8% of men and 0.5% of women see no difference between the colours red and green. These people are often useful in the military because they can detect khaki camouflage much better than people who do see a difference between red and green. What are the odds in favour of Allan being able to detect camouflage?

9. Katherine plays ringette. She has scored 4 times in 20 shots on goal. She says that the odds in favour of her scoring are 1 to 5. Is she right? Explain.

10. Jason has been awarded a penalty shot in a hockey game. Gilles is the goalie. Jason has scored 5 times in his last 10 penalty shots. Gilles has blocked 8 of the last 10 penalty shots.

a) Determine the odds in favour of Jason scoring, using his data.
b) Determine the odds in favour of Jason scoring, using Gilles' data.
c) Explain why your answers to parts a) and b) are different.

11. A survey in a Western Canadian city determined that the odds in favour of a person between 18 and 35 using a social networking site are 31 : 19. Determine the probability of a randomly selected person between 18 and 35 using a social networking site.

12. The coach of a basketball team claims that, for the next game, the odds in favour of the team winning are 3 : 2, the odds in favour of the team losing are 1 : 4, and the odds against a tie are 4 : 1. Are these odds possible? Explain.

13. Ratings for the program *Show Trial* indicate that 35% of the viewers are female, 65% are male, 30% are under 18, 20% are 19 to 30 years old, 10% are 30 to 45 years old, and 40% are older than 45. Suppose that someone is watching *Show Trial*.

a) What are the odds in favour of this person being male?
b) What are the odds in favour of this person being older than 45?

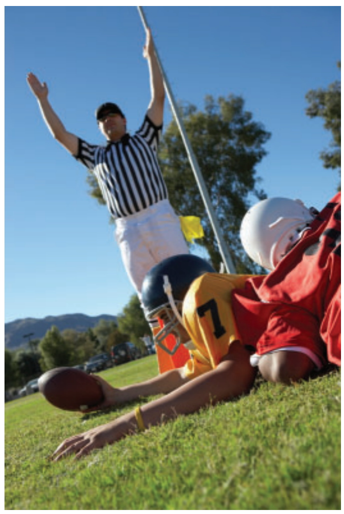
14. In a study, 70% of the people who were vaccinated did not get sick, and 42% of the people who were not vaccinated did get sick.

a) What are the odds against getting sick if you are vaccinated?
b) What are the odds against getting sick if you are not vaccinated?
c) Express the odds against from parts a) and b) with the same second term.
d) Should you be vaccinated? Explain.

15. A high-school football team has the ball at the opponent's 2 yd line. It is the third down. The team is behind by 3 points, with only one second left in the game. The players have two options:

- They can try to score a touchdown. In the past, they have succeeded 5 out of 12 times. If they score a touchdown, they will win the game.
- They can try to kick a field goal. The kicker has scored a field goal from 20 yd or less in 5 of 6 tries. If they score a field goal, they will get 3 points and tie the game, forcing overtime.

a) What are the odds in favour of each option?
b) Which option should the coach choose?



16. Three people are running for president of the student council. The polls show that Eduard Silvestre has a 45% chance of winning, Julie Jones has a 35% chance of winning, and Bill Black has a 20% chance of winning.
   a) What are the odds in favour of each person winning?
   b) Suppose that Bill Black withdraws and offers his support to Julie Jones. Further suppose that his supporters also switch to Julie Jones. What are the odds in favour of Julie winning now?

17. Grant is taking a self-study course in fitness training. He must pay $285 to take the final exam. If he fails the exam, he must pay an additional $235 to take it again. The fitness training website lists up-to-date statistics on the pass : fail ratio. The odds that a person with good study habits will pass on his or her first try are 11 : 9. Grant can prepare for the final exam by buying three practice exams for $65.
   a) Should Grant buy the practice exams if he has good study habits? Justify your opinion.
   b) If the odds in favour of passing on the first try were 17 : 4, should Grant buy the practice exams? What if the odds in favour of passing were 3 : 7? Explain.

18. a) Explain why you can express the odds against an event, A, happening as $P(A') : P(A)$.
   b) Suppose that the odds in favour of an event happening are $a : b$. Explain how you can determine the probability of the event happening. Give an example.
   c) Suppose that the probability of an event happening is $\frac{a}{c}$. Explain how you can determine the odds against the event happening. Give an example.

Closing

19. Do you prefer to express the likelihood that an event will happen using probability or odds? Explain why, and provide an example.

Extending

20. The probability that a child between the ages of 6 and 18 will need corrective lenses to see properly is 25.4%. For children between the ages of 6 and 18 who do need corrective lenses, the odds in favour of being a girl are 141 : 100.
   a) Determine the probability that a randomly selected child between the ages of 6 and 18 will be a girl who needs corrective lenses.
   b) Determine the odds that a randomly selected 6- to 18-year-old boy will need corrective lenses.