5 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 5.4, Examples 1 and 3.
- Try Chapter Review Questions 11 to 13.

Q: How can you determine if two events are mutually exclusive? Then how can you determine the probability that either of these events will occur?

A: Two events, A and B, are mutually exclusive if they cannot happen at the same time. For example, a coin can land heads or tails, but not heads and tails.

If two events are mutually exclusive, $n(A \cap B) = 0$. The favourable outcomes are represented by a pair of disjoint sets:

$$n(A \cup B) = n(A) + n(B)$$

To determine the probability that either of two mutually exclusive events will occur, add the probabilities of the two events:

$$P(A \cup B) = P(A) + P(B)$$

For example, consider these sets:
- the universal set
  $U = \{\text{natural numbers from 1 to 36, inclusive}\}$
- set A, the odd multiples of 3
  $A = \{3, 9, 15, 21, 27, 33\}$
- set B, the multiples of 4
  $B = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$

To determine the probability that a natural number chosen at random is either an odd multiple of 3 or a multiple of 4, use the following formula:

$$P(A \cup B) = P(A) + P(B)$$

Since A and B are mutually exclusive, $(A \cap B) = \{\ \}$.

$$P(A \cup B) = \frac{6}{36} + \frac{9}{36}$$
$$P(A \cup B) = \frac{15}{36}$$

The probability that a natural number chosen at random from the numbers 1 to 36 is either an odd multiple of 3 or a multiple of 4 is $\frac{15}{36}$.

Study Aid

- See Lesson 5.5, Examples 1 to 4.
- Try Chapter Review Questions 16 to 18.

Q: What is conditional probability, and how can you determine it?

A: Conditional probability occurs when two events are dependent. It means that you must consider the probability of A to determine the probability of B. This is written as $P(B \mid A)$. To determine a conditional probability, use the formula below:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

To determine the probability that two dependent events will occur, rearrange the formula as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

For example, Avery draws two cards from a standard deck of 52 playing cards, without replacing the first card. To determine the probability that both cards are hearts, let H represent the event "first card is a heart" and let R represent the event "second card is a heart":

$$P(H \cap R) = P(H) \cdot P(R \mid H)$$

$$P(H \cap R) = \frac{13}{52} \cdot \frac{12}{51}$$

$$P(H \cap R) = \frac{156}{2652}$$

The probability that the cards drawn, without replacement, are both hearts is $\frac{156}{2652}$.

Study Aid

- See Lesson 5.6, Examples 1 and 2.
- Try Chapter Review Questions 19 to 21.

Q: What are independent events, and how can you determine the probability that both will occur?

A: Two events are independent if you can determine the probability of one event without considering the probability of the other event. The following formula can be used to determine the probability that two independent events, A and B, will occur:

$$P(A \cap B) = P(A) \cdot P(B)$$

For example, determine the probability of rolling a 5 on a standard red die and a 3 on a standard blue die. The number rolled with the red die has no bearing on the number rolled with the blue die, so these events are independent.

$$P(\text{red die is 5 and blue die is 3}) = P(\text{red die is 5}) \cdot P(\text{blue die is 3})$$

$$P(\text{red die is 5 and blue die is 3}) = \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$$

The probability that a 5 is rolled on the red die and a 3 is rolled on the blue die is $\frac{1}{36}$.

PRACTISING

Lesson 5.1

1. Is each game fair or not fair? Explain.

a) Chloe and Camila each toss two coins. If all four coins land heads or tails, Chloe wins. If two coins are heads and two coins are tails, Camila wins. Otherwise, it is a tie.

b) Cooper and Alyssa take turns rolling three four-sided dice. If a 1 or a 2 is rolled at least once, then Cooper gets a point. If a 3 or a 4 is rolled at least once, then Alyssa gets a point. (Note that both could get a point on the same roll.)

2. In spinning classes, the tension of a bike is said to be 100% when the pedal can no longer be turned. Bob's spinning instructor asks him to turn the tension up to 130%. Determine the probability that Bob can do this. Explain.

Lesson 5.2

3. On National Aboriginal Day (NAD), celebrated on June 21 every year, cultural events and demonstrations take place across Canada. At a traditional Pow Wow held in Edmonton, Alberta, for NAD, 60% of the people in attendance were female. At the conclusion of the Pow Wow, everyone in attendance received a gift from the hosts. If the first person receiving a gift is chosen at random:

a) Determine the odds in favour of this person being female.

b) Determine the odds against this person being female.

4. The forecaster predicts a 70% probability of rain tomorrow. What are the odds against rain?

5. Serenity is watching Keir surf. The last seven times that Serenity watched Keir, he fell twice.

a) Determine the odds in favour of Keir falling this time.

b) Determine the odds against Keir falling this time.

6. Ariana plays sponge hockey. She has scored 6 goals in 30 shots on goal. She says that the odds against her scoring a goal are 4 : 5. Is she correct? Explain.

7. Averill is planning her university schedule. She is trying to decide between two different mathematics courses. She has been told that the odds against getting an A in the first course are 8 : 3, and the odds in favour of getting an A in the second course are 6 : 11, based on the results from previous years. Averill wants to get as high a mark as possible. Which course should she take?

Lesson 5.3

8. Two people are randomly chosen from a committee of nine people to be president and secretary. Cameron and Wyatt are on the committee. Determine the probability that they will be chosen for these roles.

9. There are 11 students, including Marina and MacKenzie, on the school swim team. The upcoming swim tournament includes a relay race. The coach has decided to choose the four positions on the relay team (first, second, third, and anchor) randomly. Determine the probability that Marina and MacKenzie will be chosen to be on the relay team.

10. Access to a particular online game is password protected. Every player must create a password that consists of three capital letters followed by four digits. For each condition below, determine the probability that a password chosen at random will contain the letters A, R, and T.

a) Repetitions are not allowed in a password.

b) Repetitions are allowed in a password.

Lesson 5.4

11. For each of the following, state whether the two events are mutually exclusive or not mutually exclusive. Explain your reasoning.
    a) Selecting a prime number or selecting an odd number from a set of 15 balls, numbered 1 to 15
    b) Rolling a sum of 8 or a sum of 6 with a pair of six-sided dice, numbered 1 to 6
    c) Eating a peach or eating an apple

12. A pinochle deck consists of 48 cards: two copies of the 9, 10, jack, queen, king, and ace cards of all four suits in a standard deck. Hunter is about to draw a card at random from a pinochle deck. If he draws a face card or a spade, he will win a point.
    a) Use A and B to represent the two favourable events. Then draw a Venn diagram to illustrate A and B.
    b) Are A and B mutually exclusive or not mutually exclusive?
    c) Determine the probability of drawing a face card or a spade.

13. The probability that Mya will exercise on Sunday is 0.5. The probability that she will go shopping on Sunday is 0.3. The probability that she will do both is 0.2.
    a) Use G and S to represent these two events. Draw a Venn diagram to illustrate G and S.
    b) Are G and S mutually exclusive or not mutually exclusive?
    c) Determine the probability that Mya will do at least one of these activities on Sunday.

14. There are 24 students in Sergio's Grade 12 class. Based on a survey he conducted, he knows that 19 of these students can swim and 8 can ski. Create a probability problem using Sergio's data. Then solve your problem, using a Venn diagram or a tree diagram.

15. Collect data on a topic that interests you. Create a probability problem using this data. Then solve your problem, using a Venn diagram or a tree diagram.

Lesson 5.5

16. Parker has eight identical black socks and 10 identical white socks loose in his drawer. He pulls out two socks at random. Determine the probability that he pulls out a pair of mismatched socks; that is, one is black and the other is white.

17. The probability that a plane will leave Winnipeg on time is 0.70. Planes that leave on time may not arrive on time. The probability that a plane will leave Winnipeg on time and arrive in Calgary on time is 0.56. Determine the probability that a plane will arrive in Calgary on time, given that it left Winnipeg on time.

18. A deck of 40 cards consists of the ace to the 10 from the four suits. A card is dealt to each of two players. Determine the probability that the first card is red and the second card is black.

Lesson 5.6

19. Peyton goes to the gym four days a week. She does a cardio workout each day, using a stair machine or a running machine or taking a salsa class. She also does a strength workout each day, using a weight machine or taking a body sculpt class. Peyton randomly chooses which cardio workout and which strength workout to do each day. Determine the probability that she will use a stair machine and take a body sculpt class the next day she goes to the gym.

20. Suppose that $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.3$. Are events A and B independent? Explain.

21. Tanya estimates that her probability of passing French is 0.7 and her probability of passing chemistry is 0.6. Determine the probability that Tanya will:
    a) Pass both French and chemistry
    b) Pass French but fail chemistry
    c) Fail both French and chemistry