6.1 Exploring the Graphs of Polynomial Functions

YOU WILL NEED

- graphing technology

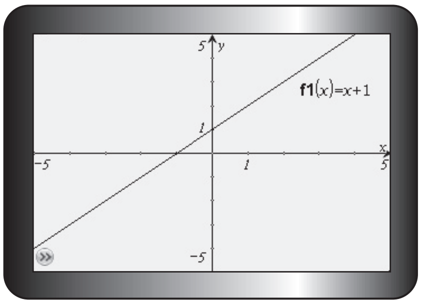
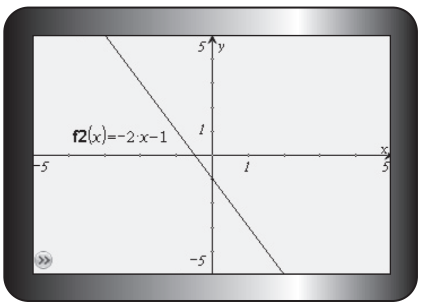
GOAL

Identify characteristics of the graphs of polynomial functions.

EXPLORE the Math

Naomi researched **polynomial functions**. She learned that they have been studied for hundreds and possibly thousands of years. Originally polynomial functions were appreciated for their simplicity, because they contain only the operations of multiplication and addition.

During her research, Naomi decided to investigate how the graphs of polynomial functions are related to the degree of the functions. She remembered many of the characteristics of linear functions and confirmed these characteristics using graphing technology.

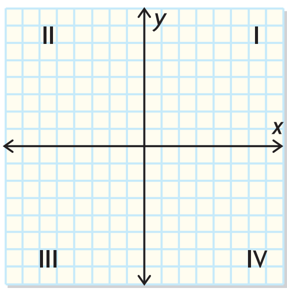
Polynomial Function	Common Characteristics
Degree 1 Graph of $f(x) = x + 1$:  Graph of $f(x) = -2x - 1$: 	**Functions of degree 1** **Number of x-intercepts:** 1 **Number of y-intercepts:** 1 **End behaviour**: Line extends either from quadrant III to quadrant I, or from quadrant II to quadrant IV. **Domain:** $\{x \mid x \in \mathrm{R}\}$ **Range:** $\{y \mid y \in \mathrm{R}\}$

end behaviour

The description of the shape of the graph, from left to right, on the coordinate plane.

Communication *Tip*

Cartesian grids are divided into four quadrants by the x-axis and y-axis. Quadrants are identified using roman numerals, from I to IV, starting from the top right and progressing counter-clockwise around the origin.



Naomi then used her graphing software to investigate the characteristics of quadratic functions and **cubic functions**. She graphed multiple examples of functions of each degree. Then she sketched some of her examples in a table and recorded her observations.

As Naomi investigated the characteristics of the graphs she created, she considered the following characteristics:

- the number of x-intercepts
- the y-intercept
- the end behaviour
- the domain
- the range

cubic function

A polynomial function of the third degree, whose greatest exponent is three; for example,

$f(x) = 5x^3 + x^2 - 4x + 1$

? What are the characteristics of the graphs of quadratic and cubic functions?

Reflecting

A. How is the possible number of x-intercepts related to the degree of a polynomial function?

B. Naomi claims that all polynomial functions of degree 1, 2, or 3 have only one y-intercept. Do you agree or disagree? Explain.

C. Describe how the end behaviour of a polynomial function is related to the degree of the function.

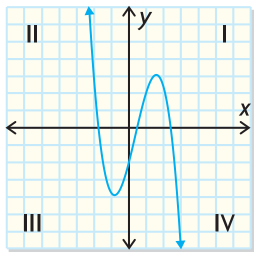
D. Describe how the domain and range of a polynomial function are related to its degree.

E. Explain why some cubic polynomial functions have **turning points** but not maximum or minimum values.

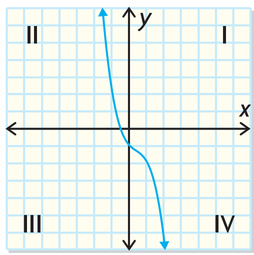
F. Polynomial functions of degree 0 are called constant functions. Describe characteristics of the graphs of constant functions.

turning point

Any point where the graph of a function changes from increasing to decreasing or from decreasing to increasing; for example, this curve has two turning points, since the y-values change from decreasing to increasing to decreasing:



This curve does not have any turning points, since the y-values are always decreasing:



Communication *Tip*

Polynomial functions are named according to their degree. Polynomial functions of degrees 0, 1, 2, and 3 are called constant, linear, quadratic, and cubic functions, respectively. The terms in a polynomial function are normally written so that the powers are in descending order. For example,

a constant function, degree **0**: $f(x) = 5x^{\mathbf{0}}$

a linear function, degree **1**: $f(x) = 2x^{\mathbf{1}} + 1$

a quadratic function, degree **2**: $f(x) = 2x^{\mathbf{2}} - x + 1$

a cubic function, degree **3**: $f(x) = 2x^{\mathbf{3}} + 3x^2 - 2x$

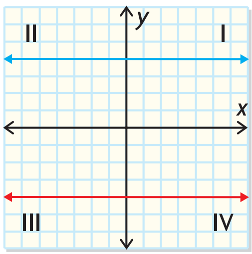
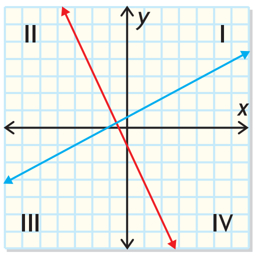
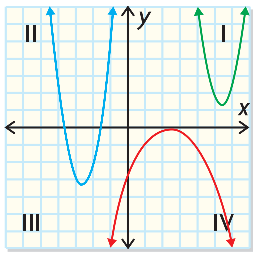
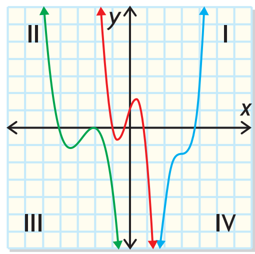
In Summary

Key Ideas

- A polynomial function in one variable is a function that contains only the operations of multiplication and addition, with real-number coefficients, whole-number exponents, and two variables. The degree of the function is the greatest exponent of the function. For example, $f(x) = 6x^3 + 3x^2 - 4x + 9$ is a cubic polynomial function of degree 3.
- The degree of a polynomial function determines the shape of the function.

Need to Know

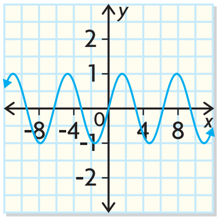
- The graphs of polynomial functions of the same degree have common characteristics.
- The chart below shows sample sketches of functions and displays all the possibilities for the x-intercepts, y-intercepts, end behaviour, range, and number of turning points for each type of function.

Type of Function	constant	linear	quadratic	cubic
Degree, *n*	0	1	2	3
Sketch				
Number of *x*-Intercepts	0, except for $y = 0$, for which every point is on the x-axis	1	0, 1, or 2	1, 2, or 3
Number of *y*-Intercepts	1	1	1	1
End Behaviour	Line extends from quadrant II to quadrant I or from quadrant III to quadrant IV.	Line extends from quadrant III to quadrant I or from quadrant II to quadrant IV.	Curve extends from quadrant II to quadrant I or from quadrant III to quadrant IV.	Curve extends from quadrant III to quadrant I or from quadrant II to quadrant IV.
Domain	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$
Range	$\{y \mid y = \text{constant}, y \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$	$\{y \mid y \leq \text{maximum}, y \in \mathrm{R}\}$ or $\{y \mid y \geq \text{minimum}, y \in \mathrm{R}\}$	$\{y \mid y \in \mathrm{R}\}$
Number of Turning Points	0	0	1	0 or 2

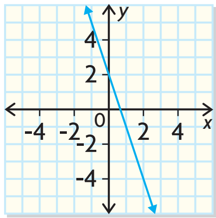
FURTHER Your Understanding

1. Determine which graphs represent polynomial functions. Explain how you decided.

a)



d)



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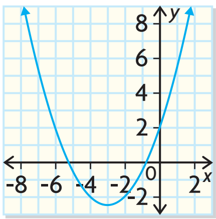

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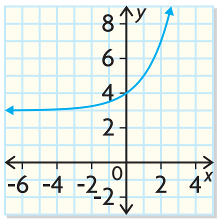

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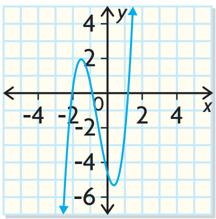
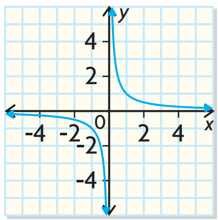


f)



2. Determine the following characteristics for each polynomial function in question 1.

- x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of turning points

3. Use technology to graph each polynomial function below. Determine the following characteristics for each function:

- number of x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of turning points

a) $f(x) = 3x - 1$

b) $f(x) = x^2 + 4$

c) $f(x) = -2x^3 - 5x + 3$

d) $f(x) = 5x^3$

e) $f(x) = -3$

f) $f(x) = -(x + 3)^2 + 2$

4. For each type of polynomial function below, sketch graphs to show all possible numbers of x-intercepts.

a) linear **b)** quadratic **c)** cubic