6 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can you tell whether a given equation represents a polynomial function?

A: A polynomial function is a function in which the coefficients are real numbers and the exponents of the variables are whole numbers. It involves only multiplication and/or addition of real numbers and variables.

For example, consider the following functions:

$$y = 3x^2 - 5x^3 + \frac{1}{2}x \qquad y = \sqrt{x} + 4x^2 - 3$$

$$y = \frac{x + 3}{2x - 5} \qquad y = \sqrt{5}x^2 + 8x - 10$$

Only two of the above equations are polynomials functions:

$$y = 3x^2 - 5x^3 + \frac{1}{2}x \quad \text{and} \quad y = \sqrt{5}x^2 + 8x - 10$$

Study Aid

- See Lesson 6.1.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can you describe the characteristics of the graph of a polynomial function by looking at the equation of the function?

A: The degree of the function, the sign of the leading coefficient, and the constant term can be used to describe the characteristics of the graph.

- The maximum number of x-intercepts of a polynomial function is equal to its degree.
- The constant term is always the y-intercept of the graph.
- If the function is linear or cubic and the leading coefficient is
  - negative, then the function extends from quadrant II to quadrant IV.
  - positive, then the function extends from quadrant III to quadrant I.
- If the function is quadratic and the leading coefficient is
  - negative, then the graph extends from quadrant III to quadrant IV.
  - positive, then the graph extends from quadrant II to quadrant I.
- All polynomial functions have the domain $\{x \mid x \in \mathrm{R}\}$.
- Linear and cubic functions have the range $\{y \mid y \in \mathrm{R}\}$.
- Quadratic functions have a range restricted by their maximum or minimum value: $\{y \mid y \geq \text{minimum}, y \in \mathrm{R}\}$ or $\{y \mid y \leq \text{maximum}, y \in \mathrm{R}\}$.
- If the function is
  - cubic, then there are no turning points or two turning points.
  - quadratic, then there is one turning point and it is either a maximum or a minimum.
  - linear, then there is no turning point; the function is a line.

Study Aid

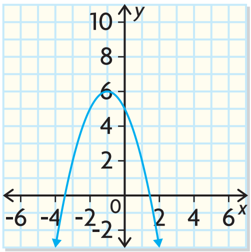
- See Lesson 6.2, Examples 1 to 3.
- Try Mid-Chapter Review Questions 3 to 6.

PRACTISING

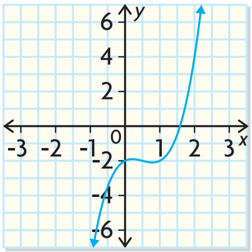
Lesson 6.1

1. Which of the following graphs might represent polynomial functions? Provide your reasoning for each graph.

a)



d)



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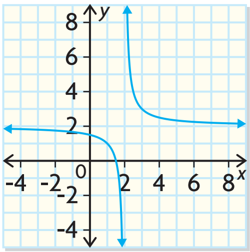

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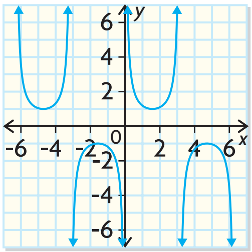

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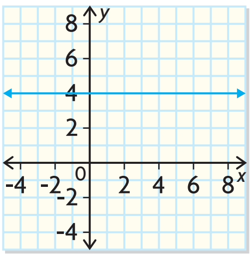

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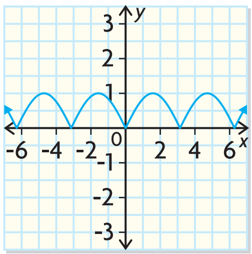


2. a) For each polynomial function in question 1, determine the degree, the domain and range, and the constant term.

b) The equations of three of the functions in question 1 are listed below. Match each equation with the correct graph, and provide your reasoning.

i) $f(x) = 2x^3 - 3x^2 + x - 2$

ii) $f(x) = 4$

iii) $f(x) = -x^2 - 2x + 5$

Lesson 6.2

3. Describe the end behaviour of each polynomial function using the degree and the leading coefficient.

a) $f(x) = -x^3 + x^2 + 10$

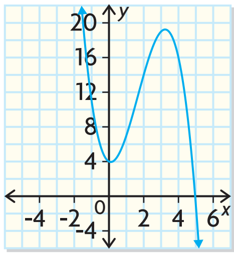
b) $f(x) = 3x(x + 2)(x - 1)$

c) $f(x) = 5x + 6$

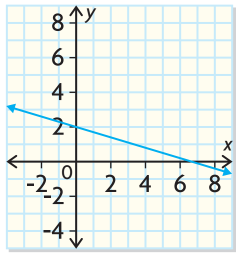
d) $f(x) = -2x^2 + 5$

4. a) Describe the characteristics of each polynomial function shown below. Include the degree, the x-intercepts, the y-intercept, the end behaviour, the domain, range, and number of turning points in your description.

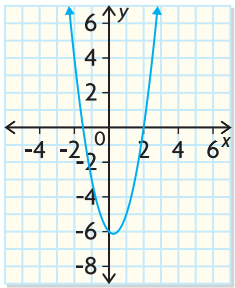
i)



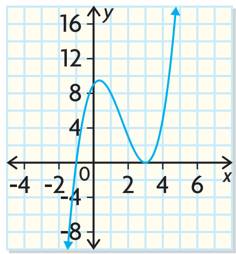
iii)



ii)



iv)



b) Determine the sign of the leading coefficient and the value of the constant term in the equation of each function.

5. Using the equation of each function, determine the possible number of x-intercepts, the y-intercept, the end behaviour, the domain, the range, and the possible number of turning points.

a) $f(x) = -(x - 2)(x + 3)$

b) $f(x) = x^3 - x^2 + x + 6$

c) $f(x) = x^2 + 5x - 1$

d) $f(x) = x^3 - 2x^2 - 8x$

6. Write a polynomial function that satisfies each set of characteristics. Use technology to check that your function satisfies the characteristics.

a) extending from quadrant III to quadrant I, two turning points, one x-intercept at the origin

b) extending from quadrant II to quadrant I, two x-intercepts

c) extending from quadrant II to quadrant IV, degree 1, y-intercept of -3

d) one turning point, one x-intercept, y-intercept of 6

e) range of $y \geq -6$, x-intercepts of 2 and 6