History | Connection

Prime-Generating Quadratic Polynomials

A prime number is a natural number that has only two divisors: 1 and itself. Prime numbers have been studied since ancient times. They are the building blocks of all numbers, since every number is either a prime number or a product of prime numbers. There are eight prime numbers under 20: 2, 3, 5, 7, 11, 13, 17, and 19.

The concept of a prime number is simple to understand, but, even today, mathematicians do not know of a fast way to determine large prime numbers. There is an infinite number of prime numbers, but there is no formula or method for coming up with a prime number.

With the advent of computing and e-commerce, large prime numbers have become increasingly important. A secure way to encrypt information, called RSA public-key encryption, requires very large prime numbers in order to keep data secure. In fact, if anyone ever determined a pattern for the primes that are used for encryption, or even a formula for producing large primes, e-commerce as we know it would crumble.

In 1772, Leonhard Euler discovered a quadratic function that produces a lot of prime numbers:

$$f(x) = x^2 + x + 41$$

This function produces 40 distinct prime numbers for all the integer values of x from 0 to 39. It also excited a lot of mathematicians with the idea that it might be possible to find a formula for all prime numbers.

Can you write a polynomial function that produces prime numbers?

The Electronic Frontier Foundation offers prizes for discovering large primes. Nayan Hajratwala was awarded $50 000 on April 6, 2000, for discovering a 1 000 000 digit prime.

A. Using technology, generate the 40 prime numbers produced by Euler's function and list them in a table.

B. Determine the value of the function when $x = 40$. How can you tell that this value is not a prime number?

C. Analyze the function to determine some of its characteristics. Why are these characteristics important for a prime-generating polynomial function?

D. Design a polynomial function that can produce at least four prime numbers.

The Clay Mathematics Institute offers $1 000 000 prizes for solving difficult mathematics problems, which the institute calls the Millennium Prize Problems. One such problem is the proof of the Riemann Hypothesis, which was formulated in 1859 by Bernhard Riemann. This proof gives a pattern for the occurrence of primes within the set of all real numbers.