7 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*
- See Lesson 7.1.
- Try Mid-Chapter Review Questions 1 and 6.

Q: How can you recognize when a graph, a table of values, or an equation represents an exponential function?

A:

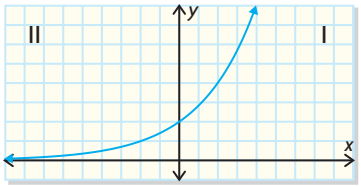
Graph

If you are given a graph, you can tell if the function is exponential by the shape of the graph. All exponential functions you have studied are one of these cases:

Case 1: an increasing curve that extends from quadrant II to quadrant I

Case 2: a decreasing curve that extends from quadrant II to quadrant I

Increasing



Decreasing

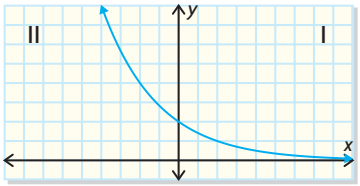


Table of values

If you are given a table of values in which the x-values increase by the same amount, calculate the ratios for consecutive pairs of y-values. If the ratios are the same, then the values represent an exponential function.

In this table of values, the y-values increase by a factor of 3. Therefore, the values represent an exponential growth function.

x	y	
0	5	$\frac{15}{5} = 3$
1	15	$\frac{45}{15} = 3$
2	45	$\frac{135}{45} = 3$
3	135	$\frac{405}{135} = 3$
4	405	$\frac{1215}{405} = 3$
5	1215	

Equation of a function

If you are given an equation, look at the form of the equation. All the exponential functions you have studied are of the form $y = a(b)^x$, where the independent variable, x, is the exponent in the equation.

Q: How are the characteristics of exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, different from the characteristics of polynomial functions of degree 3 or less?

A:

Number of x-intercepts

Exponential functions of this form have no x-intercepts. Polynomial functions of degree 3 or less may have 0, 1, 2, or 3 x-intercepts.

End behaviour

Exponential functions of this form extend from quadrant II to quadrant I. Polynomial functions of degree 3 or less can extend from quadrant III to quadrant I, quadrant II to quadrant IV, quadrant II to quadrant I, or quadrant III to quadrant IV.

Range

Exponential functions of this form have a restricted range: $\{y \mid y > 0, y \in \mathrm{R}\}$. Only polynomial functions of degree 2 have a restricted range.

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- See Lesson 7.2.
- Try Mid-Chapter Review Questions 2 to 4.

Q: How can you determine the best exponential function of the form $y = a(b)^x$ to model data?

A: Enter the given data into a graphing calculator and do an exponential regression. The exponential regression function will represent the curve of best fit for the data. You can check how well the regression function fits the data by graphing the function and the points on the same axes.

Study *Aid*

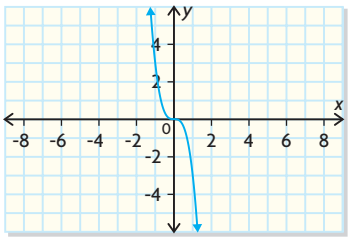
- See Lesson 7.3, Examples 1 and 2.
- Try Mid-Chapter Review Questions 8 to 11.

PRACTISING

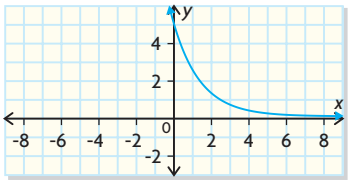
Lesson 7.1

1. Determine whether each graph represents an exponential function. Explain how you know.

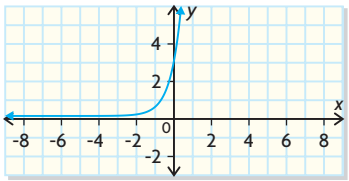
a)



b)



c)



2. a) Graph each function using technology.

i) $y = 3(4)^x$ **iii)** $y = 5(3)^x$

ii) $y = 8\left(\frac{1}{2}\right)^x$ **iv)** $y = 2\left(\frac{1}{8}\right)^x$

b) List characteristics of each function, such as the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range.

Lesson 7.2

3. State the number of x-intercepts, the y-intercept, end behaviour, domain, and range for each function, without graphing the function. Predict whether the function is increasing or decreasing. Verify your answers by graphing.

a) $y = 5(2)^x$ **b)** $y = 90\left(\frac{1}{3}\right)^x$ **c)** $y = 8e^x$

4. For each exponential function in question 1, state the following:

a) the value of a

b) whether the value of b is greater than 1 or between 0 and 1

c) the domain and range

5. Without graphing, predict which exponential functions are increasing and which are decreasing. Verify your predictions by graphing.

a) $y = 4\left(\frac{1}{3}\right)^x$ **c)** $y = 9\left(\frac{1}{2}\right)^x$

b) $y = 3(5)^x$ **d)** $y = 0.3(2)^x$

6. Lauren is president of the student council. She has designed a handout that will be distributed to the seven council members. Each of the council members will then distribute it to seven of their friends who do not have a handout, and so on, until all the students have one.

a) Create a table of values that models this situation.

b) Does the data represent an exponential function? Explain.

Lesson 7.3

7. Arneil's math teacher decided to play a dice rolling game with the students during class. The teacher gave one die to each of the 36 students. If a student got either a 1 or a 6 on a roll, the student would be out of the game. Arneil and two of his friends were the last remaining players. The data below shows how many rolls it took before they won. Determine the equation of the exponential regression function that models this data.

Number of Rolls	Number of Students Left
0	36
1	25
2	16
3	11
4	7
5	5
6	3

8 Kevin purchased his favourite hockey card for $8 in 2005. He has been tracking the value of the hockey card every year since he bought it.

Year since Purchase	Value ($)
0	8
1	13
2	22
3	40
4	72
5	124

a) Determine the equation of the exponential regression function that models this situation.

b) Use interpolation to determine the approximate value of the card:

i) 6 months after purchase

ii) 18 months after purchase

iii) 30 months after purchase

iv) 42 months after purchase

9. Paula put $1500 into a bank account that pays compound interest annually. Her bank gave her yearly figures for her investment over the next few years, as shown in the table.

Time (years)	Amount ($)
0	1500.00
1	1560.00
2	1622.40
3	1687.30
4	1754.79

a) Explain how Paula can determine the equation of the exponential growth function that models the growth of her investment.

b) What is the annual interest rate that is being paid on the account?

c) Assuming the same growth rate, how much money will Paula have in her account after 10 years?

10. Daphne has been recording the bounce heights of a ball. She determined that her data could be modelled by the exponential regression function

$$y = 140(0.80)^x$$

where y represents the height of the ball in centimetres and x represents the number of the bounce.

a) From what height was the ball first dropped?

b) Use a graph to determine the height of the ball, to the nearest centimetre, on the sixth bounce.

c) On which bounce was the height less than half the initial drop height?

11. Sonja collected data as a hot cup of coffee cooled. She determined that a cup of coffee cools according to the exponential regression function

$$C(t) = 90(0.977)^t$$

where $C(t)$ represents the temperature in degrees Celsius and t represents the time in minutes. The coffee will eventually reach a room temperature of 21 °C.

a) Without graphing, describe the characteristics of the function.

b) What are the domain and range in this context? Explain.

c) Estimate the temperature of the coffee after 10 min.

d) Use the graph of the function to determine when the coffee will reach temperatures of 60 °C and 30 °C. Round your answers to the nearest minute.

e) Estimate how long it will take for the temperature of the coffee to reach room temperature.