

Math in Action

Guess the Number: Binary Search

Suppose that you are searching for a word in a large alphabetical directory. One way to search is to start in the middle, read the word there, and then decide to look forward or backward within the directory based on whether the word you are looking for comes before or after the word you have read. You can repeat this process until you find the word you are looking for. Each time, the ordered list is split into two equal parts: one part is discarded, and the other part is searched further. This method is called a binary search, and it is often used by computers. It can be very efficient. For example, to locate one person in a list of the global population of around 7 billion, it would take a binary search algorithm just 33 steps. To find a random number between 1 and 1 000 000, it would take just 20 questions.

How many questions with a yes or no answer are required to find an unknown number from 1 to 1000 using a binary search?

- Work in pairs. Have your partner write down a secret whole number from 1 to 1000.
- To find your partner's secret number, ask questions such as the following: "Is the number greater than 500?" Record the number of questions you asked in order to find the secret number. If you say the secret number within your question, then you have found the number. For example, if the secret number is 825 and you ask, "Is the number greater than 825?" you have found the number.
- Switch roles several times, and repeat the steps above.
- On average, how many questions do you need to ask in order to find a number from 1 to 1000 using a binary search?
- Write a mathematical model that describes how many numbers remain after each question in a binary search.
- The secret number has been determined when the number of remaining numbers is 1. Solve your equation using graphing technology for the number of questions necessary to have one number remaining. How does this solution compare with the answer from your experiments?
- How many questions are required to find a secret number from 1 to 1 000 000 000 (a billion)?
- Is it possible to ask a different type of question that might be more efficient than a binary search? Test your method to see if it is more efficient.