Sine and Cosine Patterns

Recall that you have worked with **trigonometric ratios** involving angles and sides in right, acute, and obtuse triangles.

? How are the sine and cosine ratios similar, and how they are different?

A. Draw a circle with a radius of 5 cm centred at O. Draw diameter PQ and radius OY. Construct line segment YX, perpendicular to PQ at X. Your diagram should look similar to the one shown.
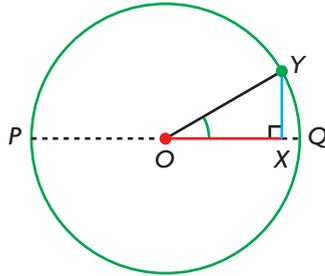
B. Measure OX, YX, $\angle YOX$, and $\angle YOQ$. Determine the sine and cosine of both angles.

C. Gradually increase the measure of $\angle YOQ$ toward 90° by moving point Y along the circumference of the circle. Repeat step B for several different angle measures.
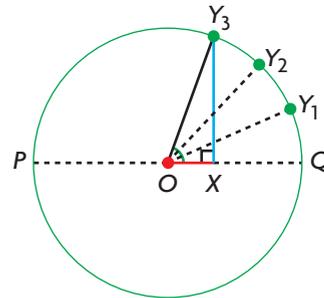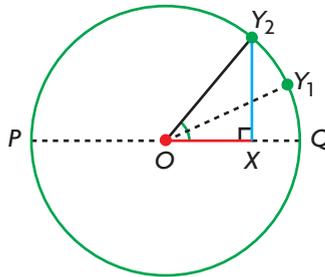
As the measure of $\angle YOQ$ increases, what happens to the following values?
- the length of YX
- the length of OX
- the value of $\sin \angle YOQ$
- the value of $\cos \angle YOQ$

D. As the measure of ∠ *YOX* gets very close to 90°, what happens to these values?
- the length of *YX*
- the value of sin ∠ *YOX*
- the length of *OX*
- the value of cos ∠ *YOX*

E. Predict the values of sin 90° and cos 90°. Provide your reasoning.

F. Move point *Y* so that the measure of ∠ *YOQ* is between 90° and 180°.
  i) How do the measures of ∠ *YOQ* and ∠ *YOX* compare?
  ii) How do the values of sin ∠ *YOQ* and sin ∠ *YOX* compare?
  iii) How do the values of cos ∠ *YOQ* and cos ∠ *YOX* compare?
  iv) What do you notice about the values of sin ∠ *YOQ* and cos ∠ *YOQ* as ∠ *YOQ* gets very close to 180°?

G. Predict the values of sin 180°, cos 180°, sin 0°, and cos 0°. Provide your reasoning.

H. Describe the pattern in the values of
- sin *x* as the measure of angle *x* increases from 0° to 180° and
- cos *x* as the measure of angle *x* increases from 0° to 180°.

I. How are the sine and cosine ratios the same?
How are they different?

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. There are many different angles that have the same sine ratio.

2. The sine ratio can never be greater than 1.

3. Functions that involve the sine and cosine ratios can be used to predict naturally occurring events.