8.2 Exploring Graphs of Periodic Functions

GOAL

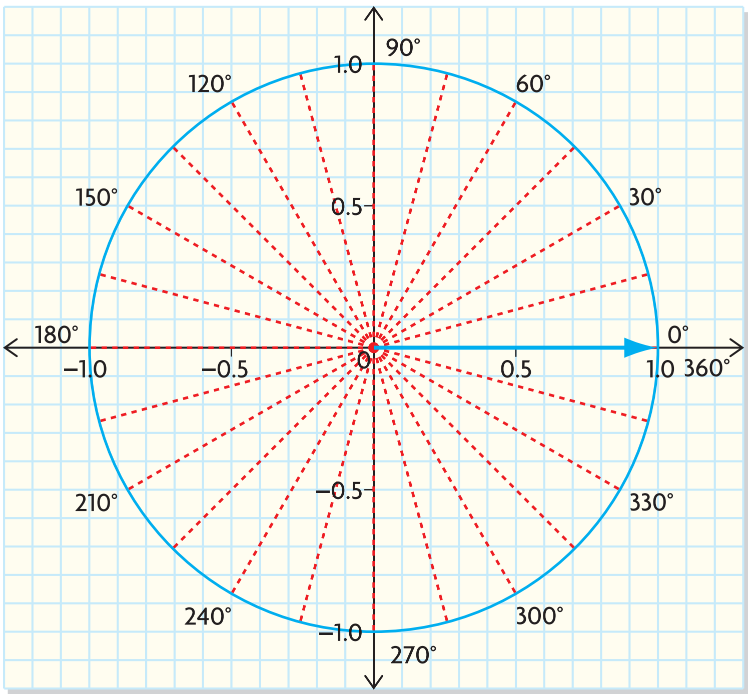
Investigate the characteristics of the graphs of sine and cosine functions.

YOU WILL NEED

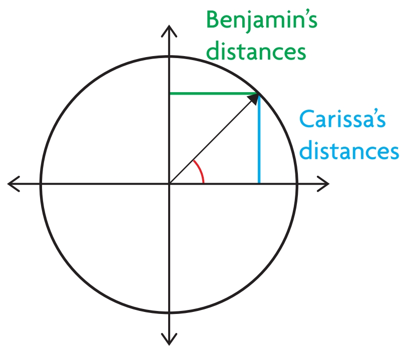
- ruler
- templates
- graphing technology

EXPLORE the Math

Carissa and Benjamin created a spinner. They glued graph paper to a board and centred the pointer at the origin. They turned the pointer counterclockwise through two complete turns.



Carissa graphed the distance from the tip of the pointer to the horizontal axis as a function of the angle, x, it had rotated. Benjamin graphed the distance from the tip of the pointer to the vertical axis as a function of the angle, x, it had rotated.



The students compared their graphs and examined:

- the number of x-intercepts
- the y-intercept
- the domain
- the range

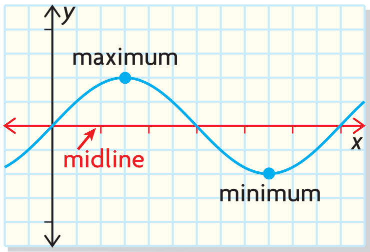
? What would Carissa's and Benjamin's graphs look like for two complete spins, and what are the characteristics of these graphs?

periodic function

A function whose graph repeats in regular intervals or cycles.

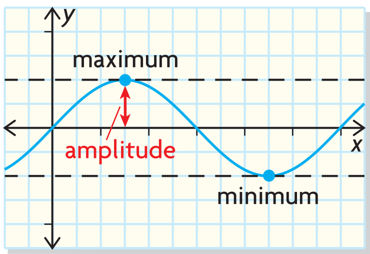
midline

The horizontal line halfway between the maximum and minimum values of a periodic function.



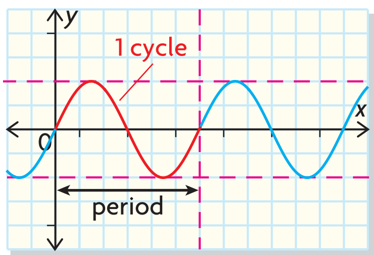
amplitude

The distance from the midline to either the maximum or minimum value of a periodic function; the amplitude is always expressed as a positive number.



period

The length of the interval of the domain to complete one cycle.



Reflecting

A. Based on your observations, which characteristics of the graphs of periodic functions are similar to characteristics of the graphs of polynomial, exponential, and logarithmic functions you have studied?

B. Based on your observations, which characteristics of the graphs of periodic functions differ from characteristics of the graphs of polynomial, exponential, and logarithmic functions you have studied?

C. Benjamin claims that if he continued drawing his graph for several more revolutions, he would notice a repeating pattern. This repeating pattern would result in a **periodic function** with several more x-intercepts. Do you agree or disagree? Explain.

D. Carissa claims that if she continued drawing her graph for several more complete revolutions, the range would remain the same. Do you agree or disagree? Explain.

E. Describe the relationships among the range, the **midline**, and the **amplitude** of each graph.

F. What is the **period** of each graph? Explain how the period helps to describe a characteristic of the graph of a periodic function.

G. Ensure that your graphing technology is in degree mode, then graph the periodic function

$$y = \sin x$$

for the domain $\{x \mid 0° \le x \le 720°, x \in \mathrm{R}\}$. How does this graph compare with Carissa's graph? Explain the reasons for any similarities or differences.

H. Graph the periodic function

$$y = \cos x$$

for the domain $\{x \mid 0° \le x \le 720°, x \in \mathrm{R}\}$ you used in part G. How does this graph compare with Benjamin's graph? Explain the reasons for any similarities or differences.

I. Ensure that your graphing technology is in radian mode, then graph

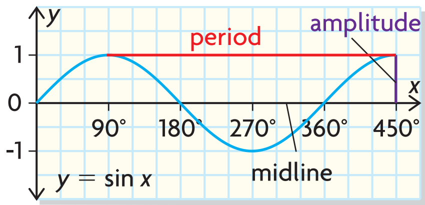
$$y = \sin x \text{ and}$$
$$y = \cos x$$

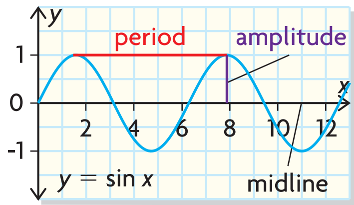
for the domain $\{x \mid 0 \le x \le 4\pi, x \in \mathrm{R}\}$. Compare these graphs with the graphs you drew in parts G and H.

In Summary

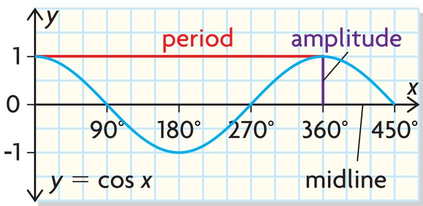
Key Ideas

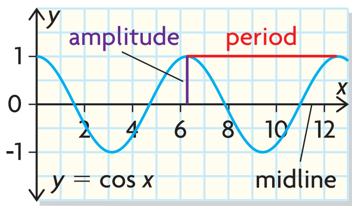
- The function $y = \sin x$ is a periodic function.





- The function $y = \cos x$ is a periodic function.

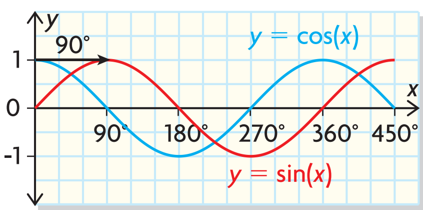


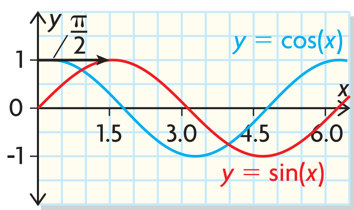


- The graphs of $y = \sin x$ and $y = \cos x$ have the following common characteristics:
  - multiple x-intercepts
  - one y-intercept
  - a domain of $\{x \mid x \in \mathrm{R}\}$
  - a range of $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$
  - an amplitude of 1
  - a period of 360° or 2π
  - a midline defined by the equation $y = 0$

Need to Know

- The graphs of $y = \sin x$ and $y = \cos x$ are congruent curves.





- The midline of the curves, $y = 0$, is the horizontal line halfway between the maximum and minimum values. The two graphs oscillate about this line.
- The period of a graph is the length of one complete cycle.

Communication Tip

Graphing technology can graph periodic functions in both degree and radian mode. Make sure your technology is in the desired mode before you enter the function and graph it.

FURTHER Your Understanding

1. a) Using graphing technology and degree measure, graph

$$y = \sin x \text{ and}$$
$$y = \cos x$$

on the same axes, for the domain $\{x \mid 0° \leq x \leq 720°, x \in R\}$.

b) What is the value of $\cos x$ when the value of $\sin x$ is a maximum? When is the value of $\cos x$ a minimum?

c) What is the value of $\sin x$ when the value of $\cos x$ is a maximum? When is the value of $\sin x$ a minimum?

d) Repeat part a) in radian measure for the domain $\{x \mid 0 \leq x \leq 4\pi, x \in R\}$.

2. The period of the function

$$y = \sin x$$

is 360° or 2π. Explain why.

3. Compare the graph of the function

$$y = \sin x$$

from 0° to 180° and from 180° to 360°. Is the graph symmetrical? If so, in what way?

4. Compare the graph of the function

$$y = \cos x$$

from 90° to 270° and from 270° to 450°. Is the graph symmetrical? If so, in what way?

5. What is the y-intercept of the graph of each function?

a) $y = \sin x$

b) $y = \cos x$

6. Determine the x-intercepts of each graph over the interval from 0° to 720°.

a) $y = \sin x$

b) $y = \cos x$

7. The graph of

$$y = \sin x$$

is sometimes called a sine wave. Discuss with a partner why this description is appropriate.

8. Adriana says that a sine graph is always a cosine graph translated left by 90°. Do you agree? Explain.

9. Suppose that Carissa put the pointer at the top of the circle and spun it counterclockwise. What would the graph representing the distance from the tip of the pointer to the horizontal axis look like? Explain.