Chapter 2 – Trigonometry

Lesson 1 – Trigonometry Review
- Page 1 – The Beauty of Triangles
- Page 2 – Pythagoras Review
- Page 3 – SOH CAH TOA Review

(No Practice Questions for this Lesson)
(No Notes for this Page)
(No Notes for this Page)

Lesson 2 – Sine Law 1
- Page 1 – Oblique Triangles
- Page 2 – Sine Law Exploration
- Page 3 – Sine Law Explained
- Page 4 – Sine Law Saves Lives

Lesson 3 – Sine Law 2
- Page 1 – Why SSA Doesn’t Always Work
- Page 2 – More SSA Explanation
- Page 3 – How Many Triangles?
- Page 4 – Finding Both Triangles

Lesson 4 – Cosine Law
- Page 1 – Why do We Need Another Law?
- Page 2 – Cooking up the Cosine Law
- Page 3 – Cosine Law Saves Lives
- Page 4 – Solving Triangles: What to use When
SOH CAH TOA Review

\[ \text{H} \rightarrow \text{hypotenuse} \quad \text{O} \rightarrow \text{opposite} \quad \text{A} \rightarrow \text{adjacent} \]

\[
\begin{align*}
\text{SOH} & \\
\sin \theta &= - \\
\text{CAH} & \\
\cos \theta &= - \\
\text{TOA} & \\
\tan \theta &= - 
\end{align*}
\]

Example:
With your spectacular math skills you get hired as an engineer at a Big Tents Incorporated. They task you to figure out how long the tie ropes should be for their new circus tent. They already know that the tent rope holds best at an angle of 62° in relation to the pole, and that the poles have to be 7 ft. How long should the rope be?
**Oblique Triangles**

An ________ is any triangle that doesn't have a ________ angle.

**Sine Law Exploration**

The ratio of the sine of angle A and the side “a”, is ________ as the ratio of the sine of angle B and the side “b”.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
Sine Law Explained

\[
\begin{align*}
\frac{\sin 77.42}{7.06} &= \\
\frac{\sin 65.19}{6.56} &= \\
\frac{\sin 37.39}{4.39} &= \\
\end{align*}
\]

**Hot Tip**
All we need to remember is that the ratio between the \( \frac{\sin A}{a} \) and \( \frac{\sin A}{a} \) is the same for every angle in the triangle.

Sine Law Saves Lives

\[
\frac{a}{\sin A} = \frac{c}{\sin C}
\]

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
Lesson 2 – Sine Law 1

Answer the questions below.

1. Challenge#

Find the length of side $x$.

![Triangle with sides 12 mm, 11.7 m, 18.2 m and angles 103°, 40°, 105°, θ.]

2. Challenge#

Find the measure of angle $\theta$.

![Triangle with sides 12 mm, 11.7 m, 18.2 m and angles 103°, 40°, 105°, θ.]

The Sine Law

When a triangle is **oblique** (no right angles), you cannot use the primary trigonometric ratios:

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{(SOH CAH TOA)}
\]

However from these ratios we can derive another tool...the **Sine Law**.

3. Consider Triangle ABC on the right. Follow the exploration below and show your work on the right...

<table>
<thead>
<tr>
<th>Exploration</th>
<th>Work space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Label the sides of the triangle ‘a, b, c’ opposite their corresponding angles.</td>
<td></td>
</tr>
</tbody>
</table>
| Draw a perpendicular down from B to AC and label it ‘h’. | ![Diagram](image)
| Write an expression for \( \sin A \) using ‘c and h’. | \( \sin A = \) |
| Write an expression for \( \sin C \) using ‘c and h’. | \( \sin C = \) |
| Rearrange each expression above to isolate ‘h’. | \( h = \) \( h = \) |
| Substitute one expression into the other and in doing so eliminate ‘h’. |           |
| Divide both sides by ‘ac’ to produce two-thirds of the Sine Law. |           |

4. Rearrange the formula to isolate ‘a’.

\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]

5. Rearrange the formula to isolate ‘sinB’.

\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]

6. Once you have isolated \( \sin B \), how do you find angle B?

**The Sine Law**
The Sine Law

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

\text{or} \quad a:b:c = \sin A: \sin B: \sin C

Use to find unknown sides and angles if you have...
- Two angles and one opposing side.
- Two sides and one opposing angle.

Solve using proportional reasoning (cross-multiply).

Find the length of the indicated side. Answer to the nearest tenth when necessary.

7. Find the length of side \(x\).

\[
\sin 40\degree = \frac{x \sin 103\degree}{12}\]

\(x = \frac{12 \sin 103\degree}{\sin 40\degree}\)

\(x = 18.2\ mm\)

8. Find the length of side \(x\).

9. Find the length of side \(x\).

10. Find the length of side \(G\).
Use the Sine Law to answer the following questions.

11. In Triangle $GHJ$,
   $\angle G = 22^\circ$, $\angle H = 86^\circ$, $g = 4.5 \text{ cm}$
   Solve the triangle.

12. In Triangle $JKL$,
   $\angle J = 127^\circ$, $\angle K = 25^\circ$, $j = 83.5 \text{ mm}$
   Solve the triangle.

13. Find the area of the following triangle to the nearest $\text{cm}^2$.

14. Find the area of the following triangle to the nearest square foot.
Find the missing angles.

15. Find the measure of angle \( \theta \). Answer to the nearest degree.

\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]

Use the Sine Law.

\[
\frac{\sin 105}{18.2} = \frac{\sin \theta}{11.7}
\]

Fill in what you know.

\[
11.7 \sin 105 = 18.2 \sin \theta
\]

Cross-multiply.

\[
\frac{11.7 \sin 105}{18.2} = \sin \theta
\]

Isolate \( \sin \theta \).

\[
\sin \theta = 0.62095
\]

Use the inverse sin function.

\[
\therefore \theta \approx 38^\circ
\]

16. Find the measure of angle \( \theta \). Answer to the nearest degree.

17. Find the measure of angle \( \theta \). Answer to the nearest degree.

18. Find the measure of angle \( \theta \). Answer to the nearest degree.

19. Sketch and solve the triangle. Answer to the nearest unit.

\( \Delta EFG, \angle G = 38^\circ, e = 18 \text{ cm}, g = 30 \text{ cm} \)

20. Sketch and solve the triangle. Answer to the nearest unit.

\( \Delta QRS, \angle R = 94^\circ, q = 85 \text{ cm}, r = 120 \text{ cm} \)
Use the Sine Law and any other tools necessary to solve the following problems.

21. Two fires are located 375 m apart on the bank of a river. A firefighter is walking along the river at point C. Find the shortest distance across the river for her to the nearest metre.

![Diagram of two fires and a firefighter on the river bank.]

22. Player A passed the puck 7 m to player B who then passed the puck 4 m to player C who then redirected the puck back to player A at an angle of 85°. What is the measure of the angle at player A to the nearest degree?

![Diagram of a hockey rink with players A, B, and C.]

23. From across a river, Calla measures the angle of elevation to a large Sequoia tree to be 74°. She then paces directly away from the tree 150 feet and measures the angle to be 50°. How tall is the tree to the nearest foot?

24. A custom shelf is to be made with three parallel shelves and a parallelogram cross-brace. The angle between the shelves and brace is 60°. The length of MQ is 28 cm and the length of RM is 30 cm. Find the length RQ.

![Diagram of a custom shelf with shelves and brace.]

25. Challenge#

Draw 2 different triangles with the following measures.

\[ \triangle ABC \text{ where } \angle A = 45°, a = 15 \text{ cm}, b = 20 \text{ cm} \]
Why SSA Doesn’t Always Work

1. What are the different number of triangles that are potentially created by different ‘a’, ‘b’ and ‘a’ values?

2. Explain in your own words why that happens.

3. How many triangles are there when ‘a’ is longer than ‘b’? Does angle ‘A’ matter then?

4. What has to be true to get....
   a. One Triangle
   b. Two Triangles
   c. No triangles

More SSA Explanation

**Question**
Why is b*sin(A) important?

[Diagram showing triangles with labels and sine calculations]

**Answer:** Because b*sin(A) is ____________________________
How Many Triangles?

Example: Given a triangle with the following measurements, how many triangles are possible?
1. In \(\triangle ABC\), \(\angle B = 72^\circ\), \(a = 23\), \(b = 31\)
   SSA
2. In \(\triangle DEF\), \(\angle E = 19^\circ\), \(e = 4\), \(f = 13\)
   SSA
3. In \(\triangle GHI\), \(\angle I = 30^\circ\), \(i = 7\), \(g = 14\)
   SSA
4. In \(\triangle GJK\), \(\angle K = 33^\circ\), \(j = 9\), \(k = 11\)
   SSA

Hot Tip
The length of the side ______ is key to discovering how many triangles are possible.

Hot Tip
There is nothing wrong with drawing the triangle with the same orientation each time.

Finding Both Triangles

![Diagram of triangles and angles]

Hot Tip
First find the _____ angle using the sine law. The obtuse angle is ________
Lesson 3 – Sine Law 2
Answer the questions below.

Draw 2 different triangles with the following measures.
\( \Delta ABC \) where \( \angle A = 45^\circ, a = 15 \text{ cm}, b = 20 \text{ cm} \)

Notice that side \( a \) has swung 39° to the left at \( \angle C \).
\( \angle A, \text{side} \ a, \text{side} \ b \) have all remained unchanged.

As you have seen previously in this chapter, supplementary angles have the same sine ratios.
Notice that the angles labeled B in the above examples are supplementary.

1. Calculate: \( \sin 70.5^\circ = \)

2. Calculate: \( \sin 109.5^\circ = \)

3. Calculate: \( \frac{\sin 70.5^\circ}{20} = \)

4. Calculate: \( \frac{\sin 109.5^\circ}{20} = \)

Since supplementary angles have the same sine ratio, it only stands to reason that dividing both by 20 also produces the same result.

This produces ambiguity in certain cases of the Sine Law.

If you are given an SSA case (2 sides and one angle), there could be one, more than one, or possibly no triangles.
Below is \( \triangle ABC \) where side ‘a’ could be placed in two different orientations.
This demonstrates the ambiguous case. Angle A, side a and side b are all unchanged in length or measure.

5. How many triangles can you create above if ...
   a. \( a = h \) ______________
   b. \( a > h \) ______________
   c. \( a < h \) ______________

8. What is the sine ratio for \( \angle A \) written in terms of \( 'h' \) and \( 'b' \)?

9. Use your ratio in the previous question to write an expression for \( h \).

10. Write a set of rules for determining how many triangles can be formed if you are given 2 sides and one opposite angle.

11. What would happen if side x was less than \( 'h' \).

12. How many triangles would be possible if side x was 51 mm long.
How many different triangles can be drawn given the following measurements?

In \( \triangle ABC \)

\[ a = 20 \]
\[ c = 16 \]
\[ \angle A = 30^\circ \]

Diagram:

Proof:

Since \( a > h \), there is the possibility for 2 triangles, however \( a > c \) also. If you were to swing side \( a \) past \( h \) towards \( \angle A \) (circular path on diagram), the triangle would not form.

\[ \therefore \text{ Only One Triangle} \]

How many different triangles can be drawn given the following measurements?

13. In \( \triangle ABC \)

\[ a = 7 \]
\[ c = 16 \]
\[ \angle A = 30^\circ \]

Diagram:

Proof:

14. In \( \triangle ABC \)

\[ a = 10 \]
\[ c = 16 \]
\[ \angle A = 30^\circ \]

Diagram:

Proof:
15. How many triangles can be formed?
\( \Delta XYZ: \angle X = 42^\circ, x = 11, z = 18 \)
Proof:
\[ h = 18 \sin 42^\circ, \ h = 12 \]
No triangle. \( x < h \)

16. How many triangles can be formed?
\( \Delta PQR: \angle P = 62^\circ, p = 32, q = 35 \)
Proof:

17. How many triangles can be formed?
\( \Delta KLM: \angle K = 30^\circ, k = 54, l = 108 \)
Proof:

18. How many triangles can be formed?
\( \Delta LNM: \angle L = 102^\circ, l = 12, n = 18 \)
Proof:

19. How many triangles can be formed?
\( \Delta TUV: \angle U = 45^\circ, u = 32, t = 22 \)
Proof:

20. How many triangles can be formed?
\( \Delta DEF: \angle D = 60^\circ, d = 100\sqrt{3}, e = 200 \)
Proof:

21. Draw a triangle with the following measurements. Then solve the triangle if possible.
\( \Delta ABC: \angle A = 105^\circ, a = 18, b = 15 \)

22. Draw a triangle with the following measurements. Then solve the triangle if possible.
\( \Delta DEF: \angle A = 105^\circ, a = 10, b = 12 \)

23. Given two sides and one opposite obtuse angle, when can a triangle be formed?

24. Given two sides and one opposite obtuse angle, when can a triangle NOT be formed?
Use the diagram below to answer the questions to the right.

25. For the triangle on the left, what value(s) of ‘a’ would produce one right triangle?

26. For the triangle on the left, what value(s) of ‘a’ would result in two unique triangles?

27. For the triangle on the left, what value(s) of ‘a’ would produce no triangles?

28. For the triangle on the left, what value(s) of ‘a’ would produce one obtuse triangle?

29. Challenge
Find the length of side x.

30. Challenge
Find the measure of $\angle C$. 
Why do We Need Another Law?

**Definition**
A bearing, in the nautical context, is a direction measured as an____ from ______, in a ________ direction.

A bearing of 10°

A bearing of 120°

Cooking up the Cosine Law

Pythagoras  Pythagoras  SOH  CAH  TOA

Cosine Law

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

1. If we know_____and________________ we can find the third side?
2. If we know all____we can find an____.
Cosine Law Saves Lives

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

Solving Triangles: What to use When

Hot Tip
Whenever we are given ___ measurements of a triangle, we can always find ______ and solve the triangle.

<table>
<thead>
<tr>
<th>Right Triangles</th>
<th>Oblique Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="right_triangle.png" alt="Right Triangle Diagram" /></td>
<td><img src="oblique_triangle.png" alt="Oblique Triangle Diagram" /></td>
</tr>
</tbody>
</table>

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Lesson 4 – Cosine Law

Answer the questions below.

Sometimes we encounter problems where we can’t use the primary trigonometric ratios or The Sine Law. We need another tool...

Triangle ABC is an oblique triangle (no right angle).

If we draw an altitude from B to point D, we create two right triangles.

To create the Cosine Law, we need to label all the parts to our diagram and consider the Pythagorean Theorem for both...

In Triangle ABD we see:

\[ h^2 + k^2 = c^2 \]

and

\[ \cos A = \frac{k}{c} \]

∴ \( k = c \cos A \)

In Triangle BCD we see:

\[ h^2 + (b - k)^2 = a^2 \]

We can use the equations above to derive the Cosine Law...

\[ h^2 + (b - k)^2 = a^2 \]
\[ h^2 + (b^2 - 2bk + k^2) = a^2 \]
\[ h^2 + k^2 + b^2 - 2bk = a^2 \]
\[ (h^2 + k^2) + b^2 - 2b(k) = a^2 \]
\[ c^2 + b^2 - 2bc \cos A = a^2 \]

1. What happened from the first line to the second line of the proof?

2. What happened from the fourth line to the fifth line of the proof?
The Cosine Law

\[ a^2 = c^2 + b^2 - 2bc\cos A \]
\[ b^2 = a^2 + c^2 - 2ac\cos B \]
\[ c^2 = a^2 + b^2 - 2ab\cos C \]

For use with non-right triangles when you know:
• 2 sides and the angle between them.
  or
• All 3 sides.

3. What patterns do you see in the three statements of the Cosine Law?

4. From the statement:
\[ a^2 = c^2 + b^2 - 2bc\cos A \]
Rearrange the formula to isolate \(\cos A\).

5. When do you feel the formula you created on the left will be useful?

6. When must you use the Cosine Law instead of the Sine Law?

7. Find the length of side \(x\).

\[ a^2 = b^2 + c^2 - 2bc\cos A \]
\[ x^2 = 450241 + 130321 - 484462\cos 120 \]
\[ x^2 = 822793 \]
\[ x = \sqrt{822793} \]
\[ x = 907 \]

8. Solve for side \(d\).
9. Find the length of side \( x \).

10. Find the length of side \( b \).

11. Find the length of side \( BC \).

12. Find the length of side \( x \).

13. Find the length of side \( x \).

14. Find the length of side \( a \).
Find the indicated angle. Round to the nearest tenth when necessary.

15. Find the measure of $\angle C$

\[ c^2 = a^2 + b^2 - 2ab\cos C \]

Rearrange cosine law to isolate $\cos C$

\[ \cos C = \frac{a^2 + b^2 - c^2}{2ab} \]

Fill in what we know.

\[ \cos A = \frac{a^2 + b^2 - 11^2}{2(8)(11)} \]

Calculate $\cos C$

\[ \cos A = 0.84659 \]

Inverse cosine ($\cos^{-1}$)

$\cos A = 32.2^\circ$

16. Find the measure of $\angle C$.

17. Find the measure of $\angle C$.

18. Find the measure of $\angle C$.

19. Find the measure of $\angle A$.

20. Find the measure of $\angle E$. 
Solve the following triangles. Answer to the nearest whole unit.

21. In $\triangle ABC$
   $AB = 17 \, \text{cm}, AC = 28 \, \text{cm}, \angle A = 36^\circ$

   ![Diagram of $\triangle ABC$]

22. In $\triangle DEF$
   $DF = 27.8 \, \text{cm}, EF = 24.7 \, \text{cm}, \angle F = 13^\circ$

   ![Diagram of $\triangle DEF$]

23. In $\triangle JKL$
   $JL = 20 \, \text{cm}, KL = 10 \, \text{cm}, \angle L = 81^\circ$

   ![Diagram of $\triangle JKL$]
True Bearing: An object’s relative position in relation to true north. We say the line pointing to the north pole is 0°.

From the origin, Point A is on a bearing of 76°.

24. Estimate the bearing of the path through B.

25. Estimate the bearing of the path through C.

26. Estimate the bearing of the path through D.

27. From the 16th tee at Cordova Bay, Tucker drives his ball 280 yards at a bearing of 40°. Todd drives his ball at 295 yards at a bearing of 75°. How far apart are the two golf balls to the nearest yard?

28. The Findlay Charles sailed at an average speed of 17 knots on a bearing of 83°. Leaving from the same port the Jack Spot travelled on average at 21 knots on a bearing of 285°. How far apart were they after one hour to the nearest kilometre?

29. Two ferrets are released from the same point on a field at exactly 1:00 pm. One feret scurries 5 m/s on a bearing of 50°. The other scampers at a speed of 6 m/s on a bearing of 220°. How far apart are they apart at exactly 1:02 pm?

30. Two students are located at the centre of the rugby field in front of the school. One student leaves at 3.2 m/s on a bearing of 150°. The other leaves at 220° at a speed of 4.3 m/s. How far apart are they after 10 seconds?
31. Leaving from base camp, a hiker travels on a bearing of $12^\circ$ at a speed of 15 km/h. After 3 hours, she changes her bearing to $42^\circ$ and travels at 12 km/h for 2 hours. How far is she from base camp at this time?

32. Two pool balls on a table are 12 cm apart. A third ball (A) is positioned as shown below. What range of angles can the ball be shot to ensure it passes between the two balls?

At many sporting events, an instrument called a total station measures horizontal throwing distance in events such as discus and javelin. The onboard computer calculates distance using the formula

$$D = \sqrt{b^2 + c^2 - 2bc\cos\theta} - R,$$

where

- $D =$ distance thrown
- $b =$ distance to impact from total station
- $c =$ distance to centre of throwing circle
- $R =$ radius of throwing circle
- $\theta =$ central angle between impact and centre of circle.

33. Total Station readings:
   - Throwing circle radius is 1.25 m
   - Distance to impact is 77.3 m.
   - Distance to Centre of circle is 60.7 m.
   - Central angle is $58.1^\circ$

   Calculate distance thrown (nearest tenth):

34. Total Station readings:
   - Throwing circle radius is 1.25 m
   - Distance to impact is 78.2 m.
   - Distance to Centre of circle is 60.7 m.
   - Central angle is $57.4^\circ$

   Calculate distance thrown (nearest tenth):
In each of the following questions, what is the most effective way to solve for the unknown?

35. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________

36. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________

37. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________

38. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________

39. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________

40. 
   a) Pythagoras Theorem  
   b) Primary Trigonometric ratios  
   c) Sine Law  
   d) Cosine Law
   Why? ____________________________________________
   __________________________________________________
   __________________________________________________
a) Pythagoras Theorem  
b) Primary Trigonometric ratios  
c) Sine Law  
d) Cosine Law

Why? ____________________________

______________________________

41. The Tower of Pisa in Italy leans to one side. Marj paces 90 metres from the base of the tower and looks up at the "high side" of the tower at 31 degrees. She knows that this side has a length of 56.70 metres. Find the angle that the tower leans at. Answer to the tenth of a degree.

42. A spotlight above a manufacturing plant floor produces a beam of light that has a beam angle of 27°. Find the area of the illuminated region if the light projects a circle on the floor. Answer to the nearest square metre.
43. An aircraft flying towards a radio tower measures the angle of depression to the tower at $5^\circ$. The pilot travels at the same bearing and altitude for 1200m and measures the angle to the tower to be $7^\circ$. How far must the pilot fly to be directly above the tower?

44. Ferguson spots an eagle’s nest in a tree in his backyard. He looks up at an angle of $30^\circ$, walks 20 metres closer to the tree and looks up at an angle of $40^\circ$ to the nest. How high is the nest in the tree?
Practice Question Answers

Lesson 2
1. 18.2 mm
2. 38°
3. \(\sin A = \frac{h}{c}, \sin C = \frac{h}{a}\)
   \(h = c \sin A, h = a \sin C\)
   \(\sin A = \frac{\sin C}{c}\)
4. \(b \sin A\)
5. \(\frac{a}{\sin B}\)
6. Inverse sine function. \(\sin^{-1}\)
7. 18.2 mm (on page)
8. 245.4 m
9. 5.8 km
10. 57.0 m
11. \(h = 12\ cm, i = 11.4\ cm, \angle L = 72°\)
12. \(k = 44.2\ mm, \quad l = 49.1\ mm, \angle L = 28°\)
13. 17 cm²
14. 5144 square feet
15. 38°
16. 70°
17. 62°
18. 37°
19. \(\angle E = 22°, \angle F = 120°, f = 42\ cm\)
20. \(\angle Q = 45°, \angle S = 41°, s = 79\ cm\)
21. 92 m
22. 35°
23. 272 feet
24. 25 cm
25. Answered on next page.

Lesson 3
1. 0.9426
2. 0.9426
3. 0.04713
4. 0.04713
5. One triangle
6. One or two. If \(a > b\) there will be only one.
7. No triangles.
8. \(\sin A = \frac{h}{b}\)
9. \(h = b \sin A\)
10. If:
   \(h < a < b: 2\ triangles\)
   \(a = h: one\ right\ triangle\)
   \(a > b: one\ triangle\)
   \(a < h: no\ triangles\)
11. There would not be a triangle formed.
12. 2 triangles would result.
13. No triangle. \(a < h\)
14. 2 triangles. \(h < a < c\)
15. No triangle. \(x < h\)
16. 2 triangles. \(h < p < q\)
17. One triangle. \(k = h\)
18. No triangle. \(l < h\)
19. One triangle. \(u > t\)
20. One triangle. \(d = h\)
21. \(\angle C = 21°, \angle B = 54°, c = 7\ cm\)
22. A triangle is not possible with these dimensions.
23. If the opposite side is longer than the other known side an obtuse triangle can be formed.
24. If the opposite side is shorter than the other given side, an obtuse triangle is not possible.
25. \(a > 216 \sin 36\)
26. \(\approx a > 127\ cm\)
27. 127 < \(a < 216\)
28. \(a < 127\)
29. 907
30. 32°

Lesson 4
1. The binomial was expanded.
2. Expressions from triangle ABD above were substituted into the equation.
3. The isolated variable corresponds to the angle used on the right side of the equation.
4. \(\cos A = \frac{a^2 - b^2 - c^2}{2bc}\)
5. When you need to find an angle.
6. The Sine Law requires you have a side and its opposite angle. The Cosine Law does not. If you have 3 sides or 2 sides and the angle between them use the Cosine Law.
7. 907
8. 11.3
9. 18.2 mm
10. 59.7 m
11. 50.2 cm
12. 32.3 miles
13. 59.2 mm
14. 3.2 km
15. 32.2°
16. 20.1°
17. 20.4°
18. 69.8°
19. 51.7°
20. 87.1°
21. $\angle B = 108^\circ, \angle C = 36^\circ,$  
    $BC = 17 \text{ cm}$
22. $\angle E = 114^\circ, \angle D = 53^\circ,$  
    $DE = 7 \text{ cm}$
23. $\angle J = 29^\circ, \angle K = 70^\circ,$  
    $JK = 21 \text{ cm}$
24. 130°
25. 240°
26. 320°
27. 173 yards
28. 37 km
29. 1315 m
30. 44 m
31. 67 km
32. The ball can be shot at a range of 10°.
33. 67.3 m
34. 67.2 m
35. Cosine Law. Given two sides and the contained angle.
37. Sine Law. Given a side and opposite angle.
38. Primary Trig ratio. Right triangle, given an acute angle and one side.
39. Primary Trig ratio. Right triangle, given an acute angle and one side.
40. Sine Law. Given a side and opposite angle.
41. 4.2 degrees to vertical.
42. 11 m²
43. 2974 m
44. 44 m